Ph.D. Macroeconomics Fall 2015
Homework Assignment #6 Due: 9:00, Thursday 26 November

1. Sticky wages and wage indexation. The economy is composed of a large number of firms distinguished by subscript *i*. Firm *i* has a production function given by $Y_i = SL_i^{\alpha}$, where *S* is a random supply shock and $0 < \alpha \leq 1$. The log of the firm's production is $y_i = s + \alpha \ell_i$, where the lower-case letters refer to the logs of the respective capital letter variables. Aggregation takes the form of averaging the logs of variables, so aggregate output is $y = s + \alpha \ell$. Aggregate demand is given by $y = m - p$. The logs of the supply shock and the aggregate-demand shock (*s* and *m*) are random variables with expected value of 0. Firms are price-takers in product markets; there is no price stickiness or imperfect competition.

a) What is the marginal product of labor? (Hint: Use the actual production function, not the logged version.) Price-taking firms will set the marginal product of labor equal to the real wage. Show that Romer's equation $p_i = w_i + (1-\alpha)\ell_i - s$ is missing a constant term. Given the algebraic expression, why is it unreasonable to assume that the omitted constant term is zero? (Note: We will ignore the constant anyway because it does not change any of the subsequent results.)
b) Suppose that labor supply is inelastic at the level of one unit per person (and per firm), so $L^s = 1$ and $\ell^s = 0$. Let aggregate labor demand be given (in log terms) by the mean across firms of Romer's price equation: $\ell^d = \frac{1}{1-\alpha}(p - w + s)$. Assume initially that the wage is perfectly flexible, in other words, that it adjusts to make aggregate labor supply equal to labor demand. Show that when $s = 0$ and $m = 0$, the equilibrium values of the logs of the endogenous variables are $w = p = y = \ell = 0$.
c) Now suppose that wages are set in contracts that last one period. At the beginning of the period, before the values of the current-period shocks are known, firms and workers set the nominal wage. Given that the expected values of the shocks are zero, they set $w = 0$ as the expected equilibrium wage. The quantity of labor employed is determined by labor demand at the fixed wage, and output by the amount that can be produced by that quantity of labor. (As part of the contracting process, workers agree to work as much as firms want them to.) Find expressions for *p, y,* and ℓ as functions of the shocks *m* and *s* and the parameter α.
d) Suppose that there is a positive demand shock in the period, $m > 0$. What happens to the equilibrium values of *y*, ℓ, and *p*? What happens to the real wage, whose log is $w - p$? Is the real wage procyclical or countercyclical?
e) Now suppose that instead of a demand shock there is a positive supply shock, $s > 0$. What happens to *y*, ℓ, and *p*? What happens to the real wage? Is the real wage procyclical or countercyclical?
f) Now consider the possibility of *indexed* contracts: instead of setting a fixed value for the nominal wage, workers and firms set a rule for determining *w* based on the value of the price level in the period. Suppose that firms and workers agree on an indexing rule stipulating that $w = \theta p$, with θ to be determined. We assume that $0 \leq \theta \leq 1$. If $\theta > 0$, this rule moves the wage away from zero whenever a demand or supply shock pushes prices

away from zero. Again, given the wage, the quantity of employment is then determined by firms' labor demand. Find expressions for p, y, and ℓ as functions of the shocks m and s and the parameters θ and α. In the previous part you examined how sensitive log price, employment, and output are to each shock when there is no indexing (θ = 0). Repeat this analysis for each shock when the wage is fully indexed (θ = 1). What happens to the elasticity of aggregate supply (the responsiveness of output to demand shocks) in the short run as the indexing parameter changes from zero to one? Explain the intuition of this result.

g) In evaluating the desirability of alternative policy rules, we often look for rules that minimize the effects of shocks. We want to find the indexing rule (*i.e.*, the value of θ) that minimizes the variance of log employment. The variance of a variable is the expected value of the squared deviation from its mean value. Thus, minimizing variance minimizes the expected squared fluctuations in log employment relative to the equilibrium level. (It is appropriate to minimize the square of log employment if the welfare loss from being away from $\ell = 0$ is measured by the area of the dead-weight loss triangle, which is proportional to the square of $\ell - 0$.)

   Standard statistical formulas for the variance of a random variable tell us that if

$$\ell = a_m m + a_s s,$$

   where the a coefficients are constants and m and s are random variables whose probability distributions have variances V_m and V_s, respectively, then

$$\text{var}(\ell) = a_m^2 V_m + a_s^2 V_s + 2a_m a_s \text{cov}(m, s),$$

   where cov(m, s) is the covariance between the two shocks. Assume that the two shocks are independent so their covariance is zero. Find an expression for the variance of employment as a function of V_m and V_s and the parameters of the model (including θ) using appropriate expressions for a_m and a_s from your analysis above. Based on this expression, find the optimal indexing rule (*i.e.*, the value of θ) that minimizes the variance of employment. Discuss optimal indexation in the limiting case of $V_m = 0$ and in the alternative limiting case of $V_s = 0$.

h) Suppose that Argentina has a high prevalence of demand shocks relative to Germany (V_m is higher in Argentina, but V_s is similar in both countries). What does this model predict about the prevalence of indexed contracts in the two countries? What does it predict about the elasticities of the short-run aggregate supply curves in the two countries? Explain the intuition behind these results.

2. Steady-state growth and inflation in the new Keynesian IS/LM model. Suppose that the economy is growing at constant rate g in the steady state and that the money supply is increasing at constant rate μ. Assume that prices are perfectly flexible.

a. Show that if the discount factor β in the utility function is $\frac{1}{1+\rho}$, then the new Keynesian IS curve (without deleting the constant term as Romer does in moving from equation (6.7) to (6.8)) can be written as $\ln Y_t = \ln Y_{t+1} - \frac{1}{\theta}(r_t - \rho)$.

b. In the steady state, $\ln Y_{t+1} - \ln Y_t = g$. What is the steady-state equilibrium value of the real interest rate? Does the LM curve affect this rate? How does the equilibrium interest rate compare to the one we derived in the Ramsey growth model?

c. In the steady state, the inflation rate is constant at an equilibrium rate π*. The LM curve can be written as $r_t = \left(\frac{M_t}{P_t}\right)^{-\nu} (Y_t)^{\theta} - \pi_{t+1}^{e}$. With expected inflation equal to the steady state, this becomes $r_t + \pi^* = \left(\frac{M_t}{P_t}\right)^{-\nu} (Y_t)^{\theta}$. Does the left-hand side of this equation change at all from year to year in the steady state? What is its "growth" rate? What is the steady-state rate of growth of the right-hand side, given that *M* grows at μ, *P* grows at π*, and *Y* grows at *g*? (Hint: Use the rules for growth rates on page 3-24 of the Coursebook.) What must the steady-state inflation rate be in order for the left-hand and right-hand sides to grow at the same rate in the steady state? Is the real money stock constant or changing over time in the steady state? Why?

d. Graphically, show how (if at all) the IS and LM curves move from year to year in the steady state and how the economy's equilibrium moves as a result. Explain what is causing each of the curves to move (or not move).

e. Consider an alternative steady state with a higher rate of money growth μ′. How would the long-run steady-state equilibrium rate of inflation, real interest rate, and nominal interest rate be different? How (if at all) would the paths of real output and the real money stock be different? Does that change in the money growth rate have any real effects? Explain.