Ph.D. Macroeconomics — Fall 2015

Homework Assignment #3 — Due: 9:00, Wednesday 21 October

1. Increase in saving with diminishing returns to produced factors. Use the model described in Romer's Section 3.3 and assume $\beta + \theta < 1$ and $n > 0$. The following equations from Section 3.3 will be useful in your answer:

- Equations for steady-state growth rates:
  - $\dot{g}_K = 0 \Rightarrow g_K^* = g_A^* + n$
  - $\dot{g}_A = 0 \Rightarrow g_K^* = \frac{\gamma n}{\beta} + \frac{1-\theta}{\beta} g_A^*$
- Equations for growth rates at any moment (steady state or transition):
  - $g_K(t) \equiv \frac{\dot{K}(t)}{K(t)} = \left[s(1-a_K)^{\alpha}(1-a_L)^{1-\alpha}\right]\left[\frac{A(t)L(t)}{K(t)}\right]^{1-\alpha}$
  - $g_A(t) = \frac{\dot{A}(t)}{A(t)} = \left[Ba_K^{\beta}a_L^{\gamma}\right]K(t)^{\beta}L(t)^{\gamma}A(t)^{\theta-1}$

Starting from a situation with the economy on its steady-state growth path, suppose that the saving rate increases.

a. Use the momentary-growth equations above to show how, if at all, the increase in the saving rate affects the growth rates of capital and knowledge at the moment of the change. Show the original $\dot{g}_A = 0$ and $\dot{g}_K = 0$ lines, the original steady-state position, and the displacement (if any) at the moment of the change on a diagram with g_K on the vertical axis and g_A on the horizontal axis.

b. Use the steady-state equations above to show how, if at all, the $\dot{g}_A = 0$ and $\dot{g}_K = 0$ lines are affected. Show the change (if any) on your graph. Given your answer, how, if at all, do the steady-state growth rates g_K^* and g_A^* change when the saving rate increases?

c. Show on your diagram how, if at all, g_K and g_A move over time from the initial point of displacement after the change to their steady-state values.

d. Has the increase in the saving rate led to a "growth effect" (a permanent increase in the growth rate) or a "level effect" (a temporary increase in growth with no change in the steady-state growth rate)? How does this compare with the results of the Solow model?

2. Simple model of learning by doing. We will analyze the following model: The production function is $Y(t) = K(t)^{\alpha}\left[A(t)\right]^{1-\alpha}$. (We assume that $L(t) = 1$ and is constant for simplicity). There is no depreciation and the saving rate is constant, so the change in the aggregate capital stock is given by $\dot{K}(t) = sY(t)$. New knowledge is created as a side-effect of producing output, so $\dot{A}(t) = BY(t)$.

a. Use the rules of growth rates (Coursebook, page 24 of chapter 3) to find expressions for $g_A(t)$ and $g_K(t)$ in terms of $A(t)$, $K(t)$, and the parameters (s, B, and α). (Neither Y nor its growth rate should appear in these equations.)

b. Find equations for the "growth rates of the growth rates" and set them equal to zero to find the conditions for $\dot{g}_K = 0$ and $\dot{g}_A = 0$. Plot these lines or curves with g_K on the vertical axis and g_A on the horizontal axis. Is there a unique steady-state growth rate for A and K in this economy? Explain. Does this model seem to correspond to Case 1 or Case 2 of Section 3.3?

c. Use the conditions for $\dot{g}_K = 0$ and $\dot{g}_A = 0$ that you derived in part b to show that the value of K/A must be constant over time for this economy on a steady-state growth path.

d. Find an equation for the unique value of K/A that satisfies the steady-state condition(s) for g_A and g_K that you derived in part b. (This will be a function of the parameters, but should not involve K, A, or Y.)

e. Plug the expression for the steady-state value of K/A from part d into the equations for $g_A(t)$ and $g_K(t)$ from part a to get an equation for the steady-state growth rates of A and K. What is the steady-state growth rate of Y?

f. Based on your result of part e, how does an increase in the saving rate affect growth in the short run or long run? Does the saving rate have a growth effect or a level effect? Is this an endogenous growth model?