Ph.D. Macroeconomics — Fall 2015

Homework Assignment #2 — Due: 9:00, Thursday 15 October

**1. Consumption and saving in the two-period model.** Suppose that a household lives for two "periods" (not necessarily years), has no initial stock of wealth, and leaves no bequest at the end of the second period. Its wage income is $W_1$ in the first period and $W_2$ in the second period. The real interest rate per period is $r$, so each unit saved in period one enables $(1 + r)$ units of consumption in period two.

a. Write the equation for the household's lifetime budget constraint as a function of $W_1$, $W_2$, $C_1$, $C_2$, and $r$.
b. How much consumption $Q_2$ would be possible in period 2 if the household consumed nothing in period 1?
c. If the household increased period 1 consumption from zero to one unit, how much would period 2 consumption have to go down?
d. Use the answers to parts b and c to draw the household's two-period budget constraint if $W_1 = 200$, $W_2 = 100$, and $r = 0.25$, with $C_2$ on the vertical axis and $C_1$ on the horizontal axis. What are the slope and vertical intercept of this line? What is the horizontal intercept?
e. Suppose that the household's lifetime utility is given by $U = \ln C_1 + \frac{1}{1+\rho}\ln C_2$. Follow the procedure of coursebook Chapter 4, Section E, to derive the slope of the household's indifference curve at the point $(C_1, C_2)$: $\left.\frac{dC_2}{dC_1}\right|_{dU=0}$. [Note: Use the formula above for the lifetime utility, replacing $u(c_t)$ with $\ln(c_t)$ and using $\frac{1}{1+\rho}$ rather than $e^{-\rho}$ as the discount factor.] Show that the slope is negative and that it increases in absolute value as $C_2$ gets larger and/or $C_1$ gets smaller, or in words, that the indifference curve bows in toward the origin.
f. Standard micro theory tells us that an interior solution for utility maximization occurs where the budget constraint is tangent to the highest indifference curve that it touches. Find the mathematical equation relating $C_2$ to $C_1$ (for given $r$ and $\rho$) where the slope of an indifference curve equals the slope of the budget constraint.
g. Use this equation to show that the household chooses a rising consumption path ($C_2 > C_1$) if and only if $r > \rho$. What is true of $C_2$ and $C_1$ if $r = \rho$? If $r < \rho$?
h. Suppose that $W_1 = 200$, $W_2 = 100$, and $\rho = 0.25$. Use the optimality condition from part f and the equation for the budget constraint to find the household's optimal $C_1$ and $C_2$ if $r = 0.25$, if $r = 0.20$, and if $r = 0.30$. Verify that your solutions reflect the properties of part g.

**2. Impact of a productivity growth slowdown.** Consider a Ramsey-Cass-Koopmans economy that is on its balanced growth path, and suppose that starting at moment $t_0$ there is a permanent decline in the rate of productivity growth: a fall in $g$.

a. How, if at all, will this affect the $\dot{k} = 0$ curve?

b. How, if at all, will it affect the $\dot{c} = 0$ curve?
c. Differentiate both sides of the $\dot{c} = 0$ equation with respect to $g$ to derive an expression for $\frac{\partial k^*}{\partial g}$ and verify that its sign corresponds to your graphical answer. Then use the $\dot{k} = 0$ equation and your answer above (remembering that $k^*$ is a function of $g$) to derive an expression for $\frac{\partial c^*}{\partial g}$ (using the chain rule) and verify that its sign corresponds to the implication of your graphical answers to parts (a) and (b). (The utility function condition in Romer's equation (2.2) is useful here.)
d. At the moment of the change $t_0$, does $c$ jump upward, jump downward, stay unchanged, or can we tell for sure? Explain.
e. Over time, what can we say about the new steady-state values to which $k$ and $c$ will converge?
f. What will happen to the long-run growth rate of $Y$ in this economy? Why?

**3. Capital taxation in the Ramsey model.** Consider a Ramsey economy that is initially on its balanced growth path. At moment $t_0$, the government begins taxing capital income at rate $\tau$. This means that each unit of capital earns only $r(t) = (1-\tau) f'(k(t))$ rather than the before-tax amount $f'(k(t))$. We want to examine the substitution effects of this tax without considering the income effects, so we initially assume that each household receives a lump-sum transfer equal to the average amount of tax collected per household. (The households will consider the transfer as fixed or exogenous because it depends on aggregate capital, not on its own capital, although in equilibrium these will be the same for the average household, so the average household's income is unchanged.)

a. Use the new expression for $r(t)$ to derive a modified Euler equation for $\dot{c} = 0$. How, if at all, does the presence of the tax affect the $\dot{c} = 0$ curve?
b. The average household's income from labor and capital earnings is now reduced to $f(k(t)) - \tau f'(k(t))$, but it also receives a lump-sum transfer equal to $\tau f'(k(t))$. Make these adjustments to income in the expression for $\dot{k}$ and show how, if at all, the presence of the tax and rebate affects the $\dot{k} = 0$ curve.
c. Show on a phase diagram the old and new steady-state equilibrium values for $c$ and $k$. Are they higher or lower?
d. Show on a phase diagram what will happen to $c$ and $k$ at the moment $t_0$ of the change. What happens to saving per effective labor unit at the moment of the change? Given the change in the incentives for saving and investment, does the change in saving make sense? Explain.
e. Show on a phase diagram the path of convergence to the new steady state. What is happening to the capital stock (per effective labor unit) during convergence and why?
f. How would your answers to parts a through c be different if instead of returning the tax revenue to consumers through lump-sum transfers the government instead used it to purchase goods and services? (Think of this as combining the analysis above with Romer's Section 2.7.)