Econ 314

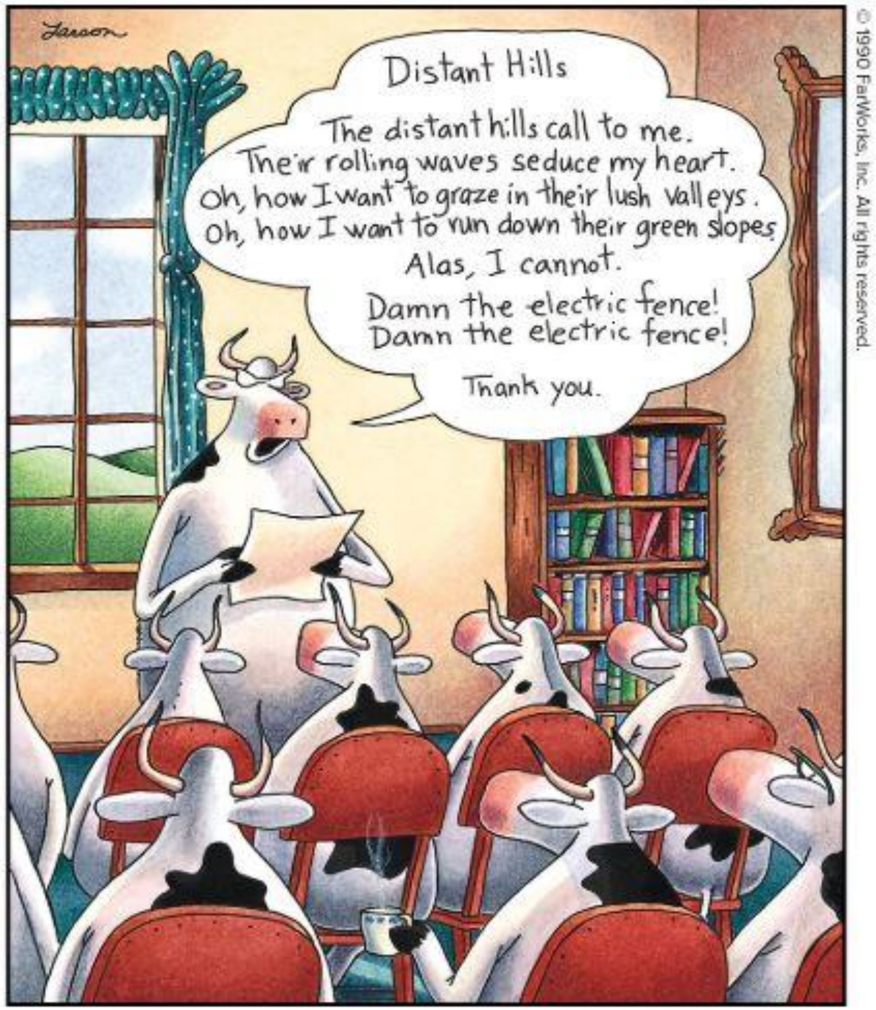
Monday, April 27

Investment with Adjustment Costs

Readings: Romer 9.2 and 9.3

Class notes: 154 - 158

Today’s Far Side offering



Cow poetry

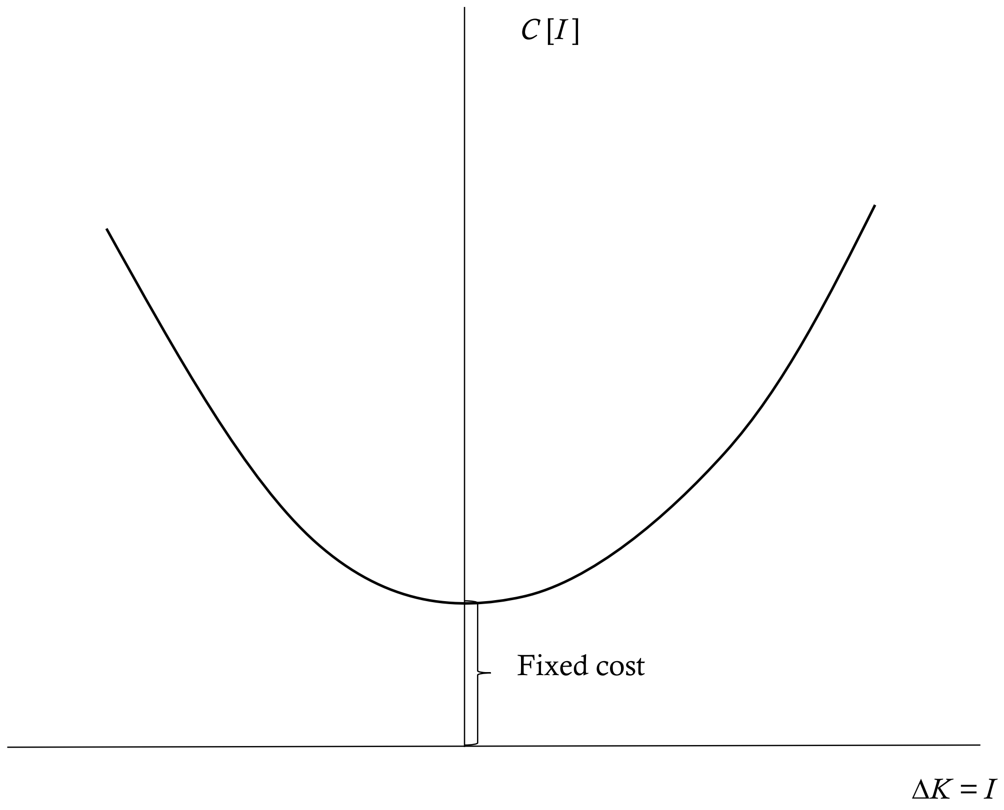
A little poetry for your cultural enlightenment…

Context and overview

- Changes in the optimal capital stock cannot cause immediate adjustment due to adjustment costs
- We model **adjustment costs** as a quadratic function of ΔK
- Firms maximize the present value of lifetime **net cash flow**
- This constrained dynamic maximization is a **Hamiltonian** problem
- The **costate variable** is q, which has useful interpretations
  - q is the **value of "installed capital"** relative to capital "on the shelf"
  - q is the **market value of the shares of stock** in a firm with one unit of capital

Fixed and variable adjustment costs

- **Fixed adjustment costs**
  - Perhaps independent of amount of investment
  - Administrative, planning, etc.
  - Fixed adjustment costs make investment "lumpy"
    - Reed adds a whole new dorm rather than one room at a time
  - We ignore fixed costs in model
- **Variable adjustment costs**
  - More costly to build quickly than more gradually
  - Variable adjustment costs cause firms to smooth investment
    - Reed takes a couple of years to build dorm rather than six months



Structure of model

- Ignore Romer's discrete-time version
- N small, price-taking firms in industry
- Representative firms has capital stock $\kappa(t)$, so industry stock is

$$K(t) \equiv N\kappa(t)$$

- "Operating profit" per unit of capital is $\pi\left[K(t)\right]$ with $\pi' < 0$
  - Larger industry capacity → supply curve shifts right and price falls
  - Firm's total operating profit is $\pi\left[K(t)\right]\kappa(t)$
- No depreciation: $\dot{\kappa}(t) = I(t)$

Modeling real adjustment costs

- Real adjustment costs depend on I, so $C = C\left[\dot{\kappa}(t)\right]$
- We assume:
  - No fixed cost: $C(0) = 0$
  - Variable costs are minimized at 0: $C'(0) = 0$, $C'' > 0$ throughout
- Simplest example is quadratic:

$$C = \frac{1}{2} a \dot{\kappa}^2$$

- We simplify Romer's analysis by using this function

Maximizing net cash flow

- Net cash flow is operating profit minus cost of new capital minus adjustment costs
  - Romer calls this "profit" but it's really net cash flow
  - Inflow and outflow of actual dollars for the firm

$$\pi\left[K(t)\right]\kappa(t)-I(t)-C\left[I(t)\right]=\pi\left[K(t)\right]\kappa(t)-I(t)-\frac{1}{2}aI(t)^2$$

- Firm seeks to maximize present value of net cash flow:

$$\max_{I(t)}\int_{t=0}^{\infty}e^{-rt}\left[\pi\left[K(t)\right]\kappa(t)-I(t)-\frac{1}{2}aI(t)^2\right]dt$$

  - Dynamically constrained because lowering *I* (good for cash flow) lowers future κ (bad for cash flow): $I(t)\equiv\dot{\kappa}(t)$

Hamiltonian

$$H\left[\kappa(t), I(t)\right] = \pi\left[K(t)\right]\kappa(t) - I(t) - \frac{1}{2}aI(t)^2 + q(t)I(t)$$

- Dynamic analogue to Lagrangian for constrained model
  - κ is the "state variable" because it cannot jump
  - *I* is the "control variable" that the firm sets at each moment
  - *q* is the "costate variable" that is a time-varying Lagrange multiplier
- In Lagrangean, the multiplier the marginal benefit from relaxing the constraint: the shadow value of income
- In Hamiltonian, the costate variable has similar interpretation: *q* is the shadow value of installed capital

First-order conditions (1)

- At every moment: $\frac{\partial H}{\partial I(t)} = 0, \ \forall t$
  - For our model $\frac{\partial H}{\partial I(t)} = -1 - aI(t) + q(t) = 0,$ or $q(t) = 1 + aI(t)$
  - Solve last expression for I to get $I(t) = \frac{1}{a}\left[q(t) - 1\right]$
  - This is essential "neoclassical investment function"
  - I is an increasing function of q
  - $q = 1$ → no desire to change capital stock
  - This is mathematically equivalent to MPK = r_K/p from last class

First-order conditions (2)

- Evolution of q: $\dfrac{\partial H}{\partial \kappa(t)} = rq(t) - \dot{q}(t)$
  - For our model $\dfrac{\partial H}{\partial \kappa(t)} = \pi\left[K(t)\right] = rq(t) - \dot{q}(t)$, or

    $\dot{q}(t) = rq(t) - \pi\left[K(t)\right]$
  - π = MPK, q = price of installed capital, $rq - \dot{q}$ is forgone interest minus capital gain on one unit of capital = user cost of capital
- Transversality condition: $\lim\limits_{t\to\infty} e^{-rt} q(t)\kappa(t) = 0$

Interpretation of q

- We can show that $q(0) = \int_{t=0}^{\infty} e^{-rt} \pi\left[K(t)\right] dt$
  - Present value of future operating profit from one unit of capital
  - π = MPK, r = user cost of capital, so q depends on MPK and user cost
  - Formally, q theory ~ to MPK = r_K/p, but with adjustment costs built in
- q is ratio of value of installed capital to uninstalled capital (1)
  - $q = 1$ → installed capital = capital on the shelf → no incentive to invest
  - $q > 1$ → installed capital is more valuable than capital on the shelf → incentive to invest because cheaper to buy off shelf than buy existing firm
- q = value of one unit of capital on stock market rather than on the shelf
- Long-run equilibrium: q adjusts to 1

Review and summary

- We set up the basics of the q theory of investment
- The Hamiltonian is the dynamic constrained maximization problem that is relevant
  - We characterized the first-order conditions for optimum
- The costate variable q has multiple interesting interpretations
  - It depends on the MPK relative to the user cost of capital
  - It represents the value or shadow price of installed capital relative to uninstalled capital
  - It is the real price of one unit of capital on the stock market

Another bad economist joke …

Three guys decide to play a round of golf: a priest, a psychologist, and an economist. They get behind a very slow twosome, who, despite having caddies, are taking all day to line up their shots and then four-putting every green. By the 8th hole, the three men are complaining loudly about the slow play ahead of them and swearing up a storm.

The priest says, "Holy Mary, I pray that they should take some lessons before they play again." The psychologist says, "I swear there are people who like to play golf slowly." The economist says, "I didn't expect to spend this much time playing a round of golf."

By the 9th hole, they have had it with slow play. The psychologist goes up to a caddie and demands that they be allowed to play through. The caddie says that would be fine, and explains that the two golfers are blind, and that both are retired firemen who lost their eyesight saving people in a fire. This explains their slow play, states the caddie. "Would you please not swear and complain so loudly?"

The priest is mortified, saying, "Here I am, a man of the cloth, and I've been swearing to the slow play of two blind men." The psychologist is also mortified, saying, "Here I am, a man trained to help others with their problems, and I've also been complaining about the slow play of two blind men."

The economist ponders the situation. He goes back to the caddies and asks, "Listen, the next time they play, could it be at night?"

--Taken from Jeff Thredgold, *On the One Hand: The Economist's Joke Book*

What’s next?

- We have all the basics of our investment theory and will now proceed to analyze the dynamics
- The equilibrium is a saddle-point equilibrium as in the Ramsey growth model
- The economy returns to a long-run equilibrium with $q = 1$
- We also consider some additional points in investment models
  - Irreversible investment
  - Empirical evidence
  - Perhaps financing issues if we have time