Econ 314

Wednesday, April 22

Search and Matching Model of Unemployment

Readings: Romer, Sections 11.4 and 11.5

Class notes: 145 - 151

Today's Far Side offering

"Mr. Osborne, may I be excused? My brain is full."

How we're all feeling at this time of year!

Context and overview

- We examine the **search/matching model** of the labor market that highlights **heterogeneity**
- Two-sided search involves **matching function** for unemployed workers and vacant jobs
- We derive the **job-finding rate** and **job-filling rate**
- Wage is set by **Nash bargaining**
- Equilibrium condition is derived with dynamic programming
- Equilibrium unemployment rate depends on:
  - Worker productivity
  - Efficiency of matching
  - Size of unemployment benefit

Model setup

- Workforce is of mass one, with share E employed and U unemployed ($E + U = 1$)
- Worker utility is: $w(t)$ if employed, b if unemployed
- Firms' job pool is filled jobs (F) and vacant jobs (V)
  - A filled job produces output y
  - All jobs (vacant or filled) cost $c < y$
  - For filled jobs: $\Pi(t) = y - w(t) - c$
  - For vacant jobs: $\Pi(t) = -c$
  - Vacancies cost nothing to create, but c to maintain
- Discount rate = r for both workers and firms

Matching function

$$M\left[U(t),V(t)\right]=k\left[U(t)\right]^{1-\gamma}\left[V(t)\right]^{\gamma}$$

- Flow from U to E … and from V to F
- **Constant returns**?
  - Thick-market externalities → increasing returns
  - Congestion externalities → decreasing returns
  - We choose middle path of constant returns
- Reverse flow is when **job matches end**: constant rate λ
- **Change in level of employment** is

$$\dot{E}(t)=M\left[U(t),V(t)\right]-\lambda E(t)$$

Rates of job finding and filling

- **Job-finding rate**: $a(t) = M[U(t), V(t)] / U(t)$
  - With CRTS, can write as

$$a(t) = m[\theta(t)] = k\theta^{\gamma}, \text{ with } \theta(t) \equiv \frac{V(t)}{U(t)} \text{ and } m[\theta(t)] \equiv M(1, \theta(t))$$

  - θ is indicator of labor-market tightness: High θ → more V relative to U → easier to find jobs
- **Job-filling rate**:

$$\alpha(t) = M[U(t), V(t)] / V(t) = \frac{m[\theta(t)]}{\theta(t)} = k\theta^{\gamma-1}$$

  - High θ → harder to fill jobs because labor is scarce

Nash bargaining

- No "equilibrium wage" because each worker/job is its own market
- **Nash bargaining** sets wage to divide up the gains to workers and firms from making a match: Share ϕ to workers and $(1 - \phi)$ to firms
- Value of ϕ depends on institutions in the economy, market conditions, etc.
- What are the **gains to each party** from the match?
  - Workers: Difference in expected lifetime utility of E vs. U
  - Firms: Difference in expected lifetime profit of F vs. V
- Use **dynamic programming** to model V_E, V_U, V_F, V_V

Applying dynamic programming

- **For a worker**:

$$rV_E(t) = w(t) - \lambda\left[V_E(t) - V_U(t)\right]$$

$$rV_U(t) = b + a(t)\left[V_E(t) - V_U(t)\right]$$

- **For a firm**:

$$rV_F(t) = \left[y - w(t) - c\right] - \lambda\left[V_F(t) - V_V(t)\right]$$

$$rV_V(t) = -c + \alpha(t)\left[V_F(t) - V_V(t)\right]$$

- Romer includes change in V but that will be zero in steady state

Steady-state equilibrium conditions

- a and α are constant values to be determined
- E is constant: $\dot{E}(t) = [U(t)]^{1-\gamma}[V(t)]^{\gamma} - \lambda E(t) = 0$
- Since vacancies are costless to create, $V_V = 0$
- Nash bargaining solution: Let total gains from match = X
  - Workers' gain = $V_E - V_U = \phi X$
  - Firms' gain = $V_F - V_V = (1-\phi)X$

$$X = \frac{V_E(t) - V_U(t)}{\phi} = \frac{V_F(t) - V_V(t)}{1-\phi}$$

$$V_E(t) - V_U(t) = \frac{\phi}{1-\phi}\left[V_F(t) - V_V(t)\right]$$

Solving for wage

- Details are in class notes, pages 148 and 149
- Wage: $$\frac{w-b}{a+\lambda+r}=\frac{\phi}{1-\phi}\frac{y-w}{\alpha+\lambda+r},$$
$$w=b+\frac{(a+\lambda+r)\phi}{\phi a+(1-\phi)\alpha+\lambda+r}(y-b)$$
- Benchmark example: $b = 0$, $a = \alpha$, and $\phi = ½$
$$w=\frac{(a+\lambda+r)\frac{1}{2}}{a+\lambda+r}y=\frac{1}{2}y$$
  - Higher ϕ, b, or a, or lower α means that workers get larger share of gains

Solving for equilibrium *a* and α

- Finding a: $\dot{E} = 0 \Rightarrow M\left[U,V\right] \equiv aU \equiv a\left(1-E\right) = \lambda E$

$$a = \frac{\lambda E}{1-E} = a\left(E\right), \text{which is increasing in } E$$

- Finding α: $\lambda E = M\left(U,V\right) = kU^{1-\gamma}V^{\gamma} = k\left(1-E\right)^{1-\gamma}V^{\gamma}$

$$V = k^{-\frac{1}{\gamma}}\left(\lambda E\right)^{\frac{1}{\gamma}}\left(1-E\right)^{\frac{\gamma-1}{\gamma}}$$

$$\alpha = M/V = k^{\frac{1}{\gamma}}\left(\lambda E\right)^{\frac{\gamma-1}{\gamma}}\left(1-E\right)^{\frac{1-\gamma}{\gamma}}$$

$$\alpha = \alpha\left(E\right) \text{ is decreasing in } E \text{ because } \gamma < 1$$

Solving for equilibrium

- Cost of creating vacancy = 0
- Value of creating vacancy must also = 0 in equilibrium

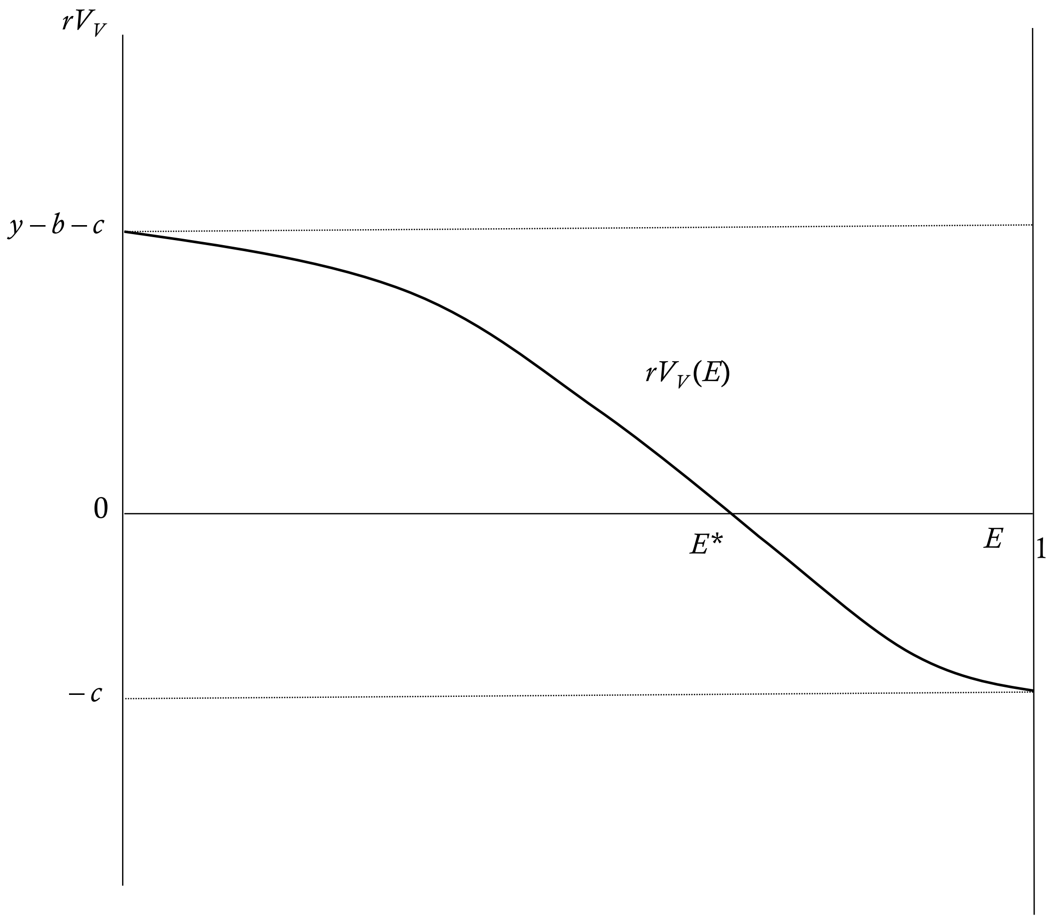
$$
\begin{aligned}
rV_V &= -c + \alpha\left[V_F - V_V\right] \\
&= -c + \alpha \frac{y - w}{\alpha + \lambda + r} \\
&= -c + \frac{(1-\phi)\alpha}{\phi a + (1-\phi)\alpha + \lambda + r}(y - b) = 0
\end{aligned}
$$

The rV_V function

$$rV_V = -c + \frac{(1-\phi)\alpha(E^*)}{\phi a(E^*) + (1-\phi)\alpha(E^*) + \lambda + r}(y-b) = 0$$

- Implicitly defines equilibrium **employment E^***
- $E = 1$ → no unemployment → $\alpha = 0$ and $rV_V = -c$
    - Value of vacancy is perpetual cost of maintaining it because job will never be filled
- $E = 0$ → $a = 0$ and $\alpha \to \infty$ → big fraction = 1 → $rV_V = y - b - c$
- Graph on next page

Determining equilibrium employment



- rV_V curve has shape shown
- Equilibrium E is at E^*
- **Effects of parameters**:
  - y↑ → curve ↑→ E^*↑
  - k↑ → curve ↑→ → E^*↑
  - b↑ → curve ↓ → E^*↓
  - Δϕ is complicated

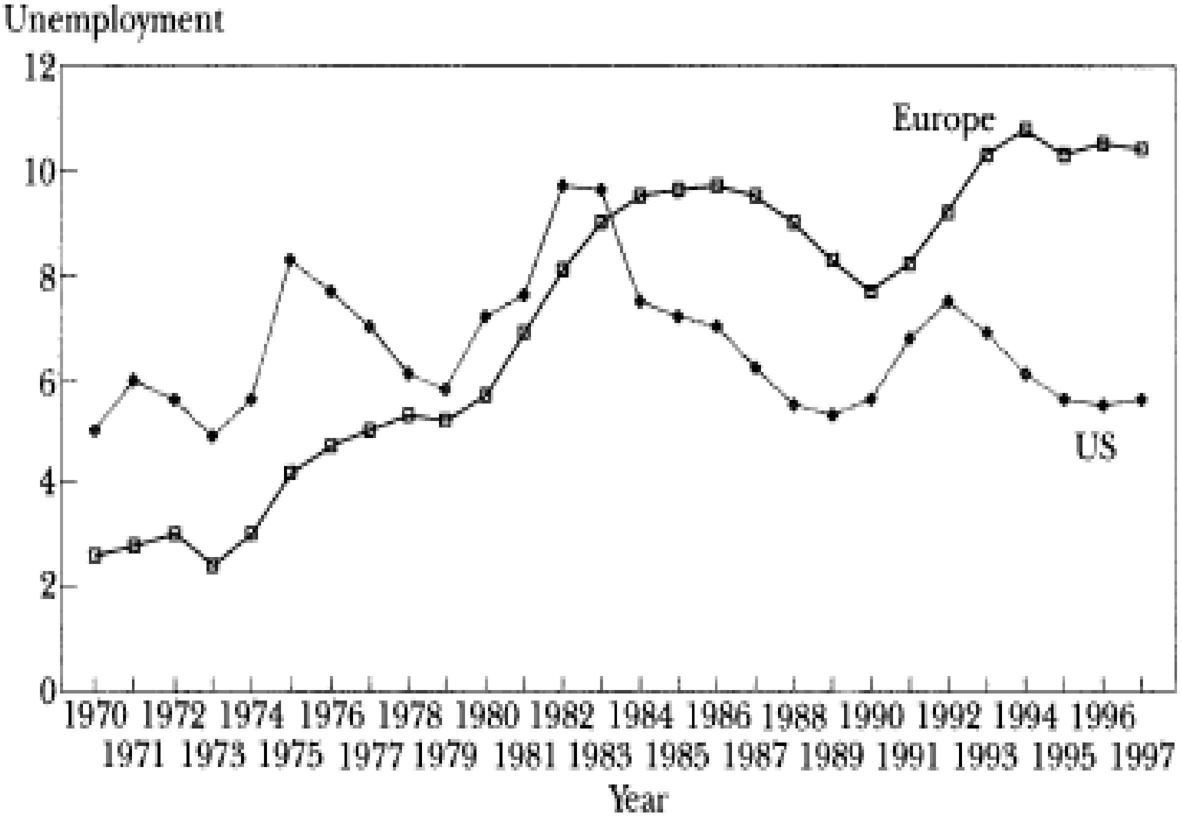
Applications

- **Sectoral shifts** (industry or geography) make it harder to make matches → k falls and unemployment increases
- **Active labor-market policies** may improve job matching → k increases and unemployment falls
  - Successful in Sweden, not so much in United States

- Caveat to the model: We are not accounting for **quality of job matches**; high-quality matches might raise utility and productivity

Empirical case study of natural rate

- US vs. Europe 1970 – 2000
- Why the changes?
- No single, simple explanation
    - Employment protection
    - Collective bargaining
    - Unemployment benefits
    - Tax rates
    - Wage flexibility
    - Labor-market flexibility



Review and summary

- Matching workers and jobs is costly
- We can model this in a Nash bargaining and dynamic programming model
- Better matching, higher productivity, and lower unemployment benefits lead to lower steady-state unemployment rate
  - More dynamic structural changes in economy make matching harder
  - Successful active labor-market policies can make matching easier
- Natural unemployment rate in Europe and U.S. diverged between 1970 and 2000, with European rates becoming very high

Another bad economist joke …

"Let us remember the unfortunate econometrician who, in one of the major functions of his system, had to use a proxy for risk and a dummy for sex."

-- Fritz Machlup

--Taken from Jeff Thredgold, *On the One Hand: The Economist's Joke Book*

What’s next?

- This concludes our discussion of unemployment
- The remaining three classes focus on business **investment** decisions
  - April 24: The nature of capital and investment
  - April 27: Modeling adjustment costs
  - April 29: Dynamics of the q theory of investment