



# *Econ 314*

**Monday, April 20**

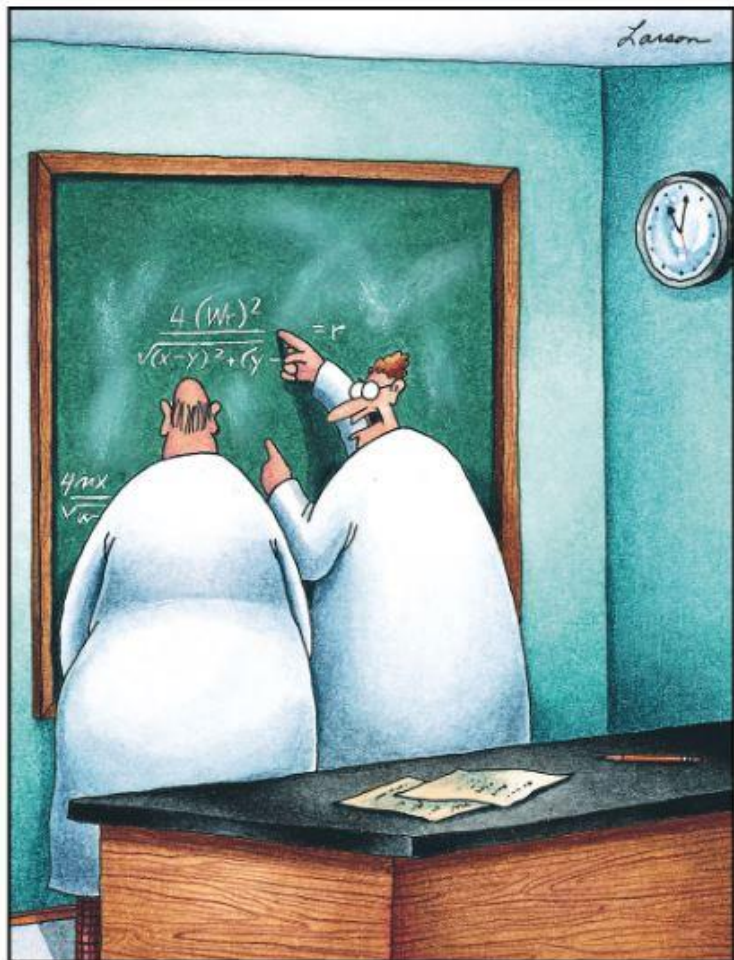
## **Solving the Shapiro-Stiglitz Model**

Readings: Romer, Section 11.2

Class notes: 142 - 145



# Today's Far Side offering



Use high-powered math  
cautiously in social  
sciences

"Yes, yes, I *know* that, Sidney—everybody knows *that!* ... But look: Four wrongs *squared*, minus two wrongs to the fourth power, divided by this formula, *do* make a right."



# Context and overview

- In the April 17 class, we set up the **Shapiro-Stiglitz model** and solved the dynamic programming problem it posed
- We concluded with three conditions that related the lifetime utility associated with each of the three states: E, S, and U
- Today, we add **equilibrium conditions** that define the equilibrium of the model
- We can summarize these conditions into two curves:
  - A **no-shirking condition** and
  - A **labor-demand curve**
- Together those curves give us equilibrium values for the real wage, the number employed, and the number unemployed



# Reviewing the lifetime utility relations

- Given the probabilities of moving between states, the values associated starting the rest of one's life in the three states are

$$\rho V_E = (w - \bar{e}) + b(V_U - V_E)$$

$$\rho V_S = w + (b + q)(V_U - V_S)$$

$$\rho V_U = a(V_E - V_U)$$

- Recall that one moves from E to U with hazard rate  $b$ , from S to U with rate  $b + q$ , and from U to E or S with rate  $a$
- One gets utility  $w$  in state S and  $w - \bar{e}$  while in E



# To work or shirk?

- Shirking hurts firms (full wage, no output) so they always pay a high-enough wage to make workers indifferent between E and S:

$$V_E = V_S$$

- We assume that if the values are equal, workers will choose work
- Since it would take only a tiny  $\varepsilon$  more to make E better than S, this does not change any of our results

$$\rho V_E = \rho V_S$$

$$w - \bar{e} - b(V_E - V_U) = w - (b + q)(V_E - V_U)$$

$$V_E - V_U = \frac{\bar{e}}{q} > 0.$$



# Solving for $w$

- From  $V_E$  equation:

$$\begin{aligned}w &= \bar{e} + \rho V_E + b(V_E - V_U) \\&= \bar{e} + (b + \rho)(V_E - V_U) + \rho V_U \\&= \bar{e} + (b + \rho + a)(V_E - V_U), \text{ because } \rho V_U = a(V_E - V_U)\end{aligned}$$

$$w = \bar{e} + (a + b + \rho) \frac{\bar{e}}{q}$$

- This is the “**no-shirking wage**”
- Firms must pay this wage to prevent workers from shirking
- Wage depends on  $\bar{e}$ ,  $q$ ,  $a$ ,  $b$ , and  $\rho$



# Steady-state equilibrium: What is $a$ ?

- In steady state, the number in each box is constant, so **flow from E to U must balance flow from U to E**
  - Number in E is  $NL$  (# of firms  $\times$  # hired per firm)
  - Number in U is  $\bar{L} - NL$
  - To balance flows:  $bNL = a(\bar{L} - NL)$

$$a = \frac{bNL}{\bar{L} - NL} \text{ and}$$

$$a + b = \frac{\bar{L}}{\bar{L} - NL} b = \frac{1}{u} b$$



# No-shirking condition

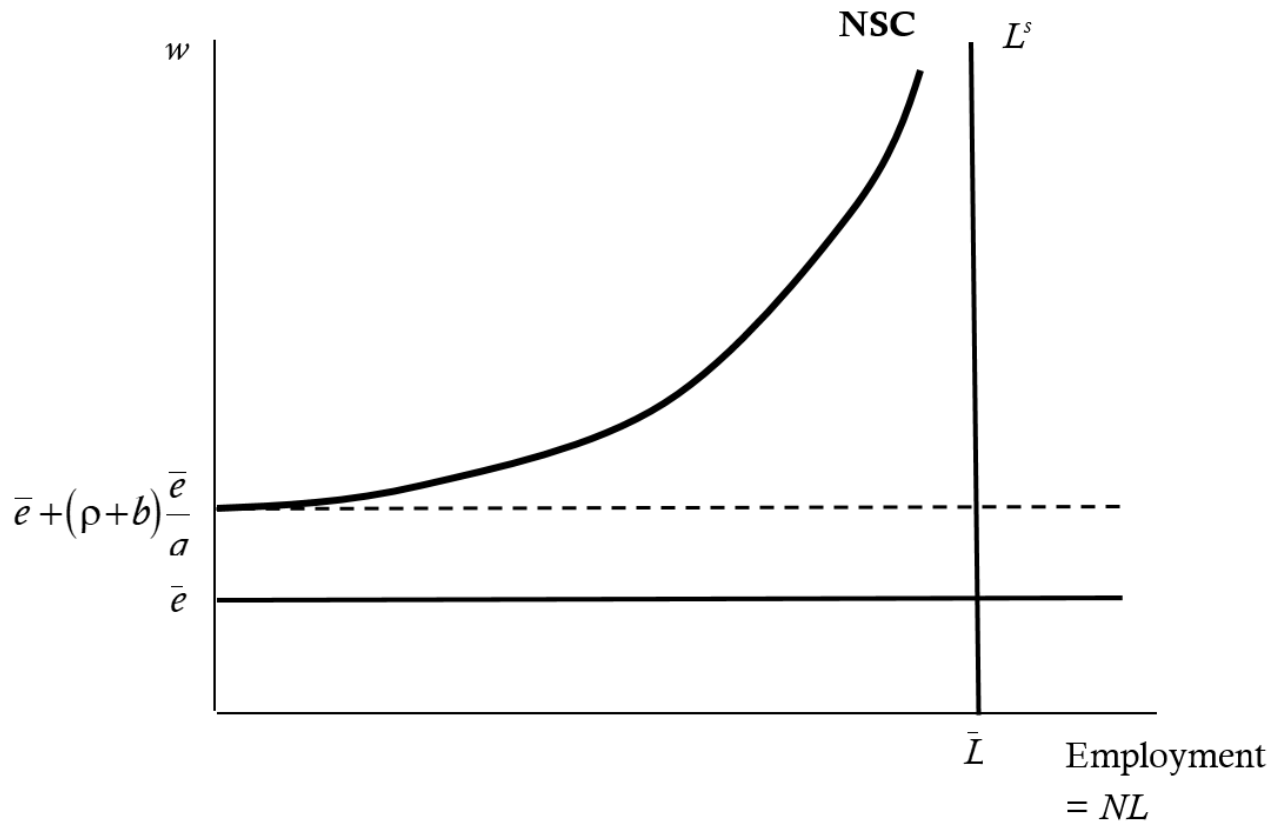
- Substituting the expression for  $a$  into the wage equation:

$$w = \bar{e} + \left( \rho + \frac{\bar{L}}{\bar{L} - NL} b \right) \frac{\bar{e}}{q}$$

- This is the **no-shirking condition** between the 2 endogenous variables  $L$  and  $w$
- It can be written with  $w$  as a linear function of  $1/u$ : a rectangular hyperbola in the unemployment rate and wage



# Graphing the no-shirking condition



- Effects

$$\bar{e} \uparrow \Rightarrow \text{NSC} \uparrow$$

$$\bar{L} \uparrow \Rightarrow \text{NSC} \rightarrow$$

$$b \uparrow \Rightarrow \text{NSC} \nearrow$$

$$q \uparrow \Rightarrow \text{NSC} \downarrow$$

- If  $q \rightarrow \infty$  then shirkers are fired in zero time and NSC is backward L
- Finite  $q$ : NSC is like labor supply curve



# Labor demand

- **Profit maximization:**

$$\Pi = F[\bar{e}L] - wL$$

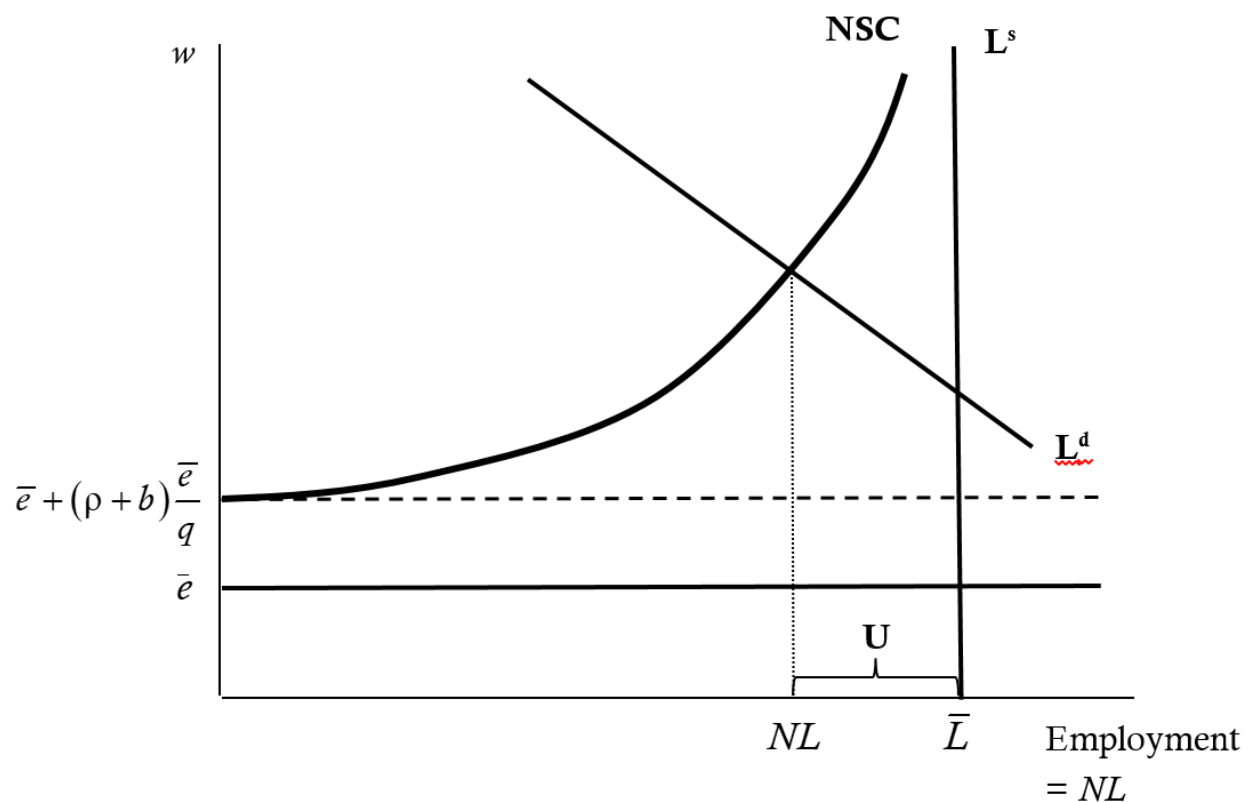
$$\frac{\partial \Pi}{\partial L} = \bar{e}F'[\bar{e}L] - w = 0$$

$$F'[\bar{e}L] = \frac{w}{\bar{e}}$$

- Negatively sloped labor-demand curve



# Labor-market equilibrium



- Perfect information about effort:  $q = \infty$  and  $L^d = L^s$
- Paying efficiency wage means **equilibrium is at  $L^d = NSC$**
- Higher wage, but positive unemployment
- “Unemployment as a worker discipline device”
- All firms pay the same, but **threat of unemployment precludes shirking**



# Issues with the model

- **Bonding**

- Employee could post bond at hiring to be forfeited if he shirks
- Would firms claim shirking and keep bond?
- Are workers sufficiently liquid to post bond up front?
- Vesting of retirement or rising wage scale over career may work this way

- **Monitoring costs**

- Would paying for better monitoring (increasing  $q$ ) be cheaper than paying efficiency wage?
  - Not for single firm because workers would go elsewhere
- Lower monitoring costs (surveillance) could raise  $q$  and lower  $w$



# Review and summary

- **Shapiro-Stiglitz model** leads to equilibrium in which firms pay higher wage to eliminate shirking
- Equilibrium involves **positive unemployment** and elevated wage
- Because all firms pay high wage, it is threat of unemployment that causes workers to work hard
- Reductions in **monitoring costs** could mitigate this problem and lower wage (and unemployment rate)



# Another bad economist joke ...

A guy walks into a Washington D.C. curio shop. After browsing, he comes across an exquisite brass rat.

“What a great gag gift,” he thinks to himself. After dickering with the shopkeeper over the price, the man purchases the rat and leaves.

As he's walking down the street, he hears scurrying sounds behind him. Stopping and looking around, he see hundreds, then thousands of rats pouring out of alleys and stairwells into the street behind him. In a panic, he runs down the street with the rats not far behind.

The street ends at a pier. He runs to the end of the pier and heaves the brass rat into the Potomac River. All of the rats scurry past him into the river, where they drown.

After breathing a sigh of relief and wiping his brow, the man heads back to the curio shop, finds the shopkeeper, and asks, “Do you have any brass economists?”

--Taken from Jeff Thredgold, *On the One Hand: The Economist's Joke Book*



# What's next?

- The next two classes (April 22 and 24) study the **search and matching** model of the labor market
- This model relies on **heterogeneity of workers and jobs** to motivate the need for a matching function that matches up unemployed workers with vacant jobs
- We again use dynamic programming to model movement between states