Econ 314

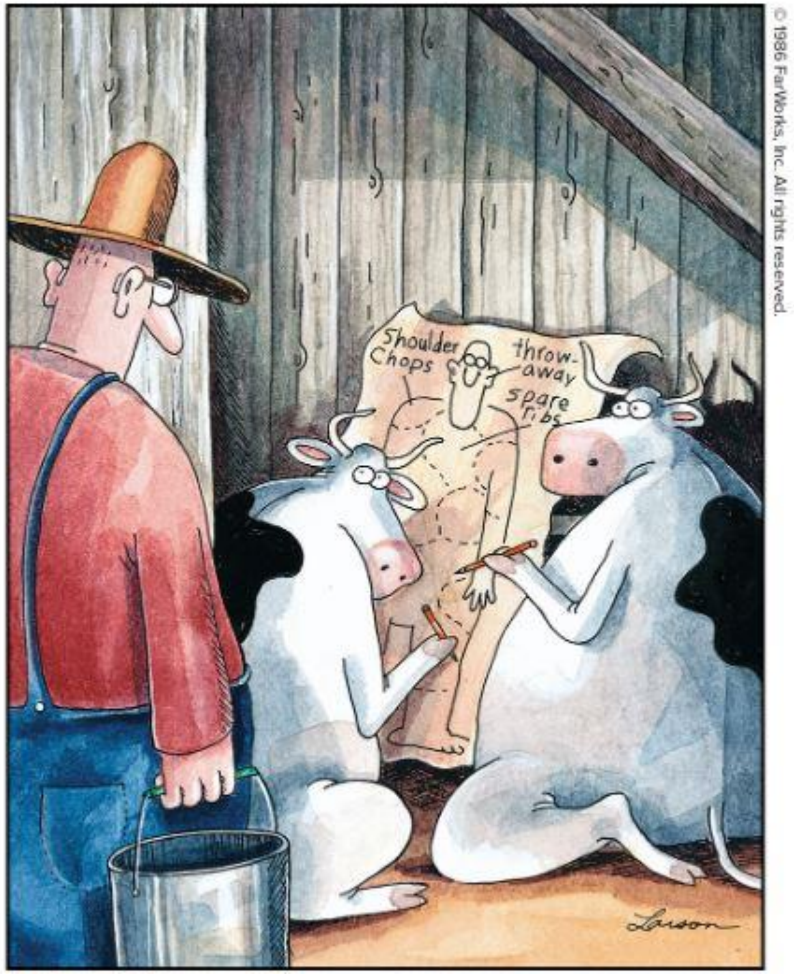
Friday, April 19

Dynamic Programming and the Setup of the Shapiro-Stiglitz Model

Readings: Romer, Section 11.2, Coursebook, Chapter 14, pages 26-31

Class notes: 138 - 142

Today’s Far Side offering



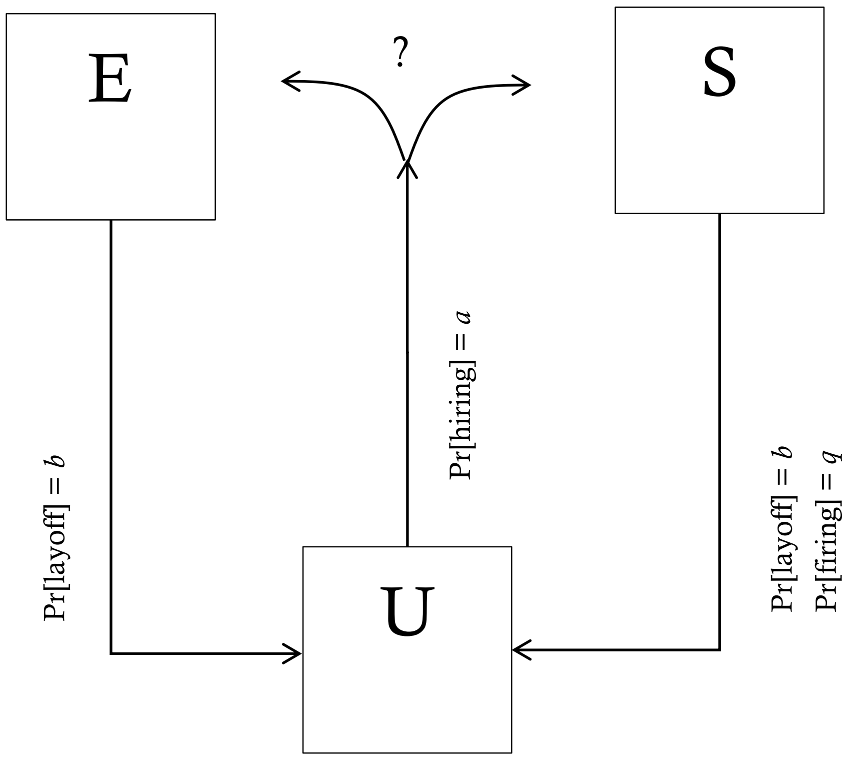
Farmer Brown froze in his tracks; the cows stared wide-eyed back at him. Somewhere, off in the distance, a dog barked.

Hmmmm

Context and overview

- The **Shapiro-Stiglitz model** has workers in one of three states: employed and working hard, employed but shirking, and unemployed
- We use **dynamic programming** to analyze movements (flows) between states over time
- **Hazard rates** are the annualized probabilities of instantaneous movement from one state to another
- The **Bellman equation** expresses the value of being in state X at time *t* as the sum of the flow of current utility in state X and the expectation of the value of each state in the future, weighted by the probability of moving/staying there
- We can express the relationships among the values associated with each state as a simple function of the flow of utility gained in that state, the values of the other states, and the hazard rates of changing states

States in Shapiro-Stiglitz model



- E = employed and working hard
- S = employed but shirking
- U = unemployed

- Hazard rates:
  - E → U: b
  - S → U: $b + q$
  - U → E/S: a

Hazard rates

- **Hazard rates** are like probabilities, but in continuous time
- Workers can change states at any instant, but we have to measure probabilities over finite time periods such as a year
  - Example: could be laid off at any moment, but what is the probability that you would be laid off over the course of a year?
- We consider this as the limiting case of discrete change times
- Suppose layoffs only happen at end of year and b is annual probability of layoff:
  - Probability of not having been laid off after one year = $(1-b)^1$
  - What happens if we extend this to multiple layoff dates per year?

Hazard rates as limit of probabilities

- Layoffs 2 times per year (middle and end with probability ½*b*):
  - Probability of not being laid off at end of year = $\left(1-\frac{1}{2}b\right)^2$
- Four times per year? "Survival" probability = $\left(1-\frac{1}{4}b\right)^4$
- Once a day? Probability = $\left(1-\frac{1}{365}b\right)^{365}$
- At any moment: Probability of surviving one year = $\lim_{n\to\infty}\left(1-\frac{1}{n}b\right)^n = e^{-b}$
- If b is hazard rate of moving out of state E, then probability that someone starting in E at $t = 0$ still being in E Δt periods later is $e^{-b\Delta t}$
- For state S = $e^{-(b+q)\Delta t}$ and for state U = $e^{-a\Delta t}$

Utility and profit

- **Instantaneous utility** in the three states:

$$u(t)=\begin{cases} w(t)-\bar{e} \text{ if working hard (E)} \\ w(t) \text{ if employed but shirking (S)} \\ 0 \text{ if unemployed} \end{cases}$$

- **Lifetime utility**: $U=\int_0^\infty e^{-\rho t}u(t)dt$

- **Firms' profit**: $\Pi(t)=F\left(\bar{e}L(t)\right)-w(t)\left[L(t)+S(t)\right]$

Dynamic programming: Bellman equation

- **Expected lifetime utility** of someone currently in state i = $V_i(0)$

$$V_i(0) = \lim_{\Delta t \to 0} V_i(\Delta t) \equiv \lim_{\Delta t \to 0} \left\{ \int_{t=0}^{\Delta t} u(t\,|\,\text{state at } 0 = i)\,dt + e^{-\rho \Delta t} E\left[V(\Delta t\,|\,\text{state at } 0 = i) \right] \right\}$$

- For state E:

Discount factor for time *t*

Probability she's employed at *t*

Momentary utility of being employed

Probability she's unemployed at *t*

Momentary utility of being unemployed

$$V_E(\Delta t) = \int_{t=0}^{\Delta t} e^{-\rho t} \left[e^{-bt} (w - \bar{e}) + (1 - e^{-bt}) (0) \right] dt$$

$$+ e^{-\rho \Delta t} \left[e^{-b\Delta t} V_E(\Delta t) + (1 - e^{-b\Delta t}) V_U(\Delta t) \right]$$

Future value of being unemployed at Δt

Discount factor for time Δt

Probability she's employed at Δt

Future value of being employed at Δt

Probability she's unemployed at Δt

Working with Bellman equation

$$\int_{t=0}^{\Delta t} e^{-(\rho+b)t}\left(w-\overline{e}\right)dt = \left[-\frac{\left(w-\overline{e}\right)}{\rho+b}e^{-(\rho+b)\Delta t}\right]-\left[-\frac{\left(w-\overline{e}\right)}{\rho+b}e^{-(\rho+b)0}\right]$$

$$= \left[-\frac{w-\overline{e}}{\rho+b}e^{-(\rho+b)\Delta t}\right]+\frac{w-\overline{e}}{\rho+b} = \left(1-e^{-(\rho+b)\Delta t}\right)\frac{w-\overline{e}}{\rho+b}$$

- Substituting into Bellman equation, collecting terms, and taking limit as $\Delta t \to 0$, we get

$$V_E = \frac{w-\overline{e}+bV_U}{\rho+b} \quad \text{or} \quad \rho V_E = \left(w-\overline{e}\right)+b\left(V_U - V_E\right)$$

Interpretation

$$\rho V_E = (w - \bar{e}) + b(V_U - V_E)$$

- Left side = flow "**utility return**" on being in E
  - "Interest rate" ρ times "asset value" V_E
- First term on right = flow utility "**dividend**" received while in E
- Last term on right = "**expected capital gain/loss**" from moving from E
  - Probability of moving = b
  - Change in asset value from move = $V_U - V_E < 0$

Bellman equations for E, S, and U

- Applying this logic to all three states allows us to write the Bellman equations directly without doing integrals and limits:

$$\rho V_E = (w - \overline{e}) + b(V_U - V_E)$$

$$\rho V_S = w + (b + q)(V_U - V_S)$$

$$\rho V_U = a(V_E - V_U)$$

- These are three equations in the three values V_E, V_S, and V_U
  - If we knew w and a, we could solve them
  - We will need some additional assumptions about the labor market to provide equations to get w and a (next class)

Review and summary

- The **Shapiro-Stiglitz model** considers workers' incentives to shirk, or slack off on the job
- We analyze the model using **dynamic programming**, modeling utility in a world in which workers can be in any of three states: **employed** and working hard, employed and **shirking**, or **unemployed**
- Shirking gives the highest momentary utility, but has a higher risk of firing, which makes the worker unemployed (lowest momentary utility)
- We can characterize lifetime utility by the **Bellman equation**, which relates the value of being in a state to the utility return in the state and the expected change from moving to another state

Something different

If these were normal times, I'd be inviting you to come and see our marimba band perform on Saturday at our teacher/leader's annual concert.

This year, the best I can do is offer a clip from last year's performance, recorded on our old camcorder with crappy sound (even worse when compressed here!) and people walking in front of the camera.

What's next?

- In the next class (April 20), we use today's analysis to derive the equilibrium in the Shapiro-Stiglitz model
  - What is the wage?
  - What is the level of employment?
  - What is the level of unemployment?
  - How to these variables respond to the parameters of the model?

- This gives us our first theory of unemployment, based on firms paying efficiency wages in excess of the market-clearing wage in order to motivate workers not to shirk