Econ 314

Solving the Imperfect-Competition Model

Monday, March 30, 2020

Reading: Romer Chapter 6, pp. 272-275

Class notes: Pages 102 to 105

Daily problem: #27

Today’s Far Side offering

Context and overview

- **Last class**: We derived the demand equation for good *i* in the imperfect competition model: $C_i = \left(\frac{P_i}{P}\right)^{-\eta} C$

- **Today**: We examine **labor supply and demand**, the **supply of goods** by firms, and solve for the general **market equilibrium** to characterize the behavior of output, prices, real wages, and employment

Household’s labor decision

- We made the utility function additive in labor and consumption so that these decisions can be analyzed separately: $U = C - \frac{1}{\gamma} L^{\gamma}$
- No saving, so $C = \frac{WL + R}{P}$, where R is the household’s nominal income from firms’ profits
- Plugging into utility function: $U = \frac{WL + R}{P} - \frac{1}{\gamma} L^{\gamma}$

Labor supply

- Maximizing $U = \frac{WL + R}{P} - \frac{1}{\gamma} L^{\gamma}$ with respect to L:

$$\frac{\partial U}{\partial L} = \frac{W}{P} - L^{\gamma - 1} = 0$$

$$\boxed{L = \left(\frac{W}{P} \right)^{\frac{1}{\gamma - 1}}}$$

- **Labor supply** is upward-sloping function of W/P

Aggregation of output

- Recall that we defined C to be a Dixit-Stiglitz aggregation across the continuum of goods:

$$C = \left(\int_{i=0}^{1} C_i^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}$$

- We use the same aggregator to create an **index of output**:

$$Y = \left(\int_{i=0}^{1} Y_i^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

Demand and profit

- In equilibrium, $C_i = Y_i$ (and $C = Y$), demand for good i is $Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y$
- **Real profit** for i is $\frac{R_i}{P} = \frac{P_i Y_i - WL_i}{P} = \left(\frac{P_i}{P}\right) Y_i - \left(\frac{W}{P}\right) L_i$

- W and P are given; firm chooses P_i/P, Y_i, and L_i subject to
  - **Production function**: $Y_i = L_i$
  - **Demand for good *i***: $Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y$

Pricing for profit maximization

- Substituting: $\frac{R_i}{P} = \left(\frac{P_i}{P}\right)^{1-\eta} Y - \left(\frac{W}{P}\right)\left(\frac{P_i}{P}\right)^{-\eta} Y$

- Choosing relative price to maximize real profit:

$$\frac{\partial\left(R_i / P\right)}{\partial\left(P_i / P\right)} = (1-\eta)\left(\frac{P_i}{P}\right)^{-\eta} Y + \eta\left(\frac{W}{P}\right)\left(\frac{P_i}{P}\right)^{-\eta-1} Y = 0$$

- We can simplify this to

$$\frac{P_i}{P} = \frac{\eta}{\eta-1}\left(\frac{W}{P}\right)$$

Characterizing pricing decision

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1}\left(\frac{W}{P}\right)$$

- Firm chooses relative price to be markup on marginal cost W/P
- **Markup ratio** is $\eta/(\eta - 1) > 1$
  - This is standard result from monopoly theory that you may have studied in Econ 201
- As demand becomes more elastic, firms are more competitive and $\eta \to \infty$
  - Markup ratio goes to 1 and competitive firms price at marginal cost

Aggregate demand

- As in Lucas model, we use M as a general index of the level of **aggregate demand** = nominal spending, so $PY = M$
- This M corresponds to the nominal spending variable S that we used in our consumption model
- In deriving consumption: $C = S/P$, but $Y = C$, so

$$Y = \frac{M}{P}$$

- This is our "theory" of aggregate demand

Assumption of symmetry

- Solution of all of the equations is straightforward except we need to assume that all firms in the model are identical
  - All have same demand elasticity η
  - All have same production function
  - All face the same real wage W/P
- Because of this, they all choose the same P_i/P
- But ***P* is aggregate average** of P_i, so all $P_i = P$ and firms choose

$$\frac{P_i}{P} = 1$$

Equilibrium

- Simple algebra (shown on page 105 of notes) gets us to

$$Y = \left(\frac{\eta - 1}{\eta}\right)^{\frac{1}{\gamma - 1}} < 1$$

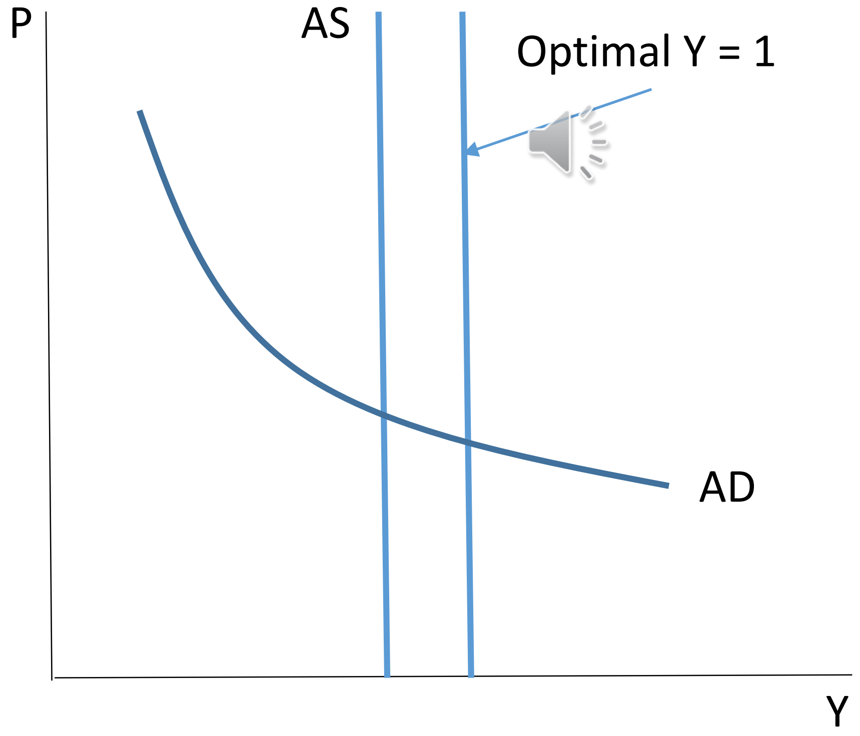
$$P = \frac{M}{Y} = \frac{M}{\left(\frac{\eta - 1}{\eta}\right)^{\frac{1}{\gamma - 1}}}$$

Properties of equilibrium

- **Inefficient output and employment**
  - When we derived the optimal properties in a Robinson Crusoe setting we had optimal $Y = L = 1$
  - The imperfect-competition model leads to $\boldsymbol{Y < 1}$ **and** $\boldsymbol{L < 1}$
  - Monopolies produce too little
  - If equilibrium output is suboptimal, then booms in output above the natural level are good!
- **Money is neutral**
  - Increase in M increases P proportionally, but doesn't affect Y or L

Aggregate supply and demand

Because it is not affected by AD, the aggregate supply curve is vertical, but it lies to the left of the optimal level of output



Review and summary

- We examined several behavioral decisions
  - Households' labor supply decision
  - Firms' profit-maximizing pricing decision
- We then solved the model using the symmetry of firms to aggregate prices
- Two main conclusions:
  - Equilibrium (natural) output and employment are below optimal levels
  - Aggregate demand (money) has no real effects as long as there is no price stickiness

From *The Devil's Dictionary*

Distance, *n*. The only thing that the rich are willing for the poor to call theirs, and keep.

[Note that for Ambrose Bierce, as well as for me, "distance" is a NOUN, not a VERB. I refuse to engage in "social distancing," though I am perfectly willing to be "keeping social distance"!]

What's next?

- In the next class (April 1), we will discuss the phenomenon of "**coordination failures**" as presented in a seminal 1988 paper by Russell Cooper and Andrew John