Econ 314

Monday, March 16

Setting up the imperfect competition model

Key Points

- New Keynesians wanted a model of price stickiness with solid microfoundations
- Price-takers in competitive markets cannot keep prices sticky
- We build a model based on monopolistic competition, to which we will later add sticky prices
- For today, we just analyze utility maximization and the demand for each good i as a function of aggregate demand and the price of i
- In the next class, we shall analyze production and equilibrium in the model

Firms and market structure

- There are many, small firms in the economy
  - Sometimes modeled as $i = 1, 2, \ldots, N$ with large N
  - We take the limit as $N \to \infty$ to get a "continuum of firms" indexed by $i \in [0,1]$
- Each firm's product is a close (but not perfect) substitute for those of ALL other firms, with demand elasticity $1 < \eta < \infty$
  - Higher η means more competition; it is infinite in perfect competition
- No capital, investment, government, or foreign sector so aggregate $Y = C$.

Production function

- We choose the simplest possible production function to illustrate the properties of the model: $Y_i = L_i$
  - We don't need capital, technology, or diminishing returns to labor, so keep everything as simple as possible

Utility function

- We need utility function with consumption and labor effort:

$$U = C - \frac{1}{\gamma} L^{\gamma}$$

- Additivity keeps things simple
- $\gamma > 1$ assures that marginal disutility of labor is rising

Optimal behavior with no imperfection

- Household in isolation, which consumes its own product
- What would be optimal work effort $\bar{L}$?
- $C = L$ so

$$U = \bar{L} - \frac{1}{\gamma}\bar{L}^{\gamma}$$

$$\frac{\partial U}{\partial \bar{L}} = 1 - \bar{L}^{\gamma-1} = 0$$

$$\bar{L} = 1, \ \bar{Y} = C = 1$$

Consumption with infinite variety of goods

- C must reflect consumption of ALL goods
- We use an aggregate index of consumption that averages across goods

Dixit-Stiglitz index with N goods

$$C = \left(\sum_{i=1}^{N} C_i^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

Infinite variety of goods

- We let $N \to \infty$ to make the impact of each good go to zero
- Goods are now indexed by $i \in [0, 1]$

Dixit-Stiglitz index with infinite variety

$$C = \left(\int_{i=0}^{1} C_i^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}$$

Maximizing utility as two-step process

- Romer does utility maximization in two steps:

$$S \equiv \int_{i=0}^{1} P_i C_i \, di$$

1. Given total nominal spending S, how does a household decide how much of each good to buy?

2. How is S determined?

Demand for C_i given S

- Lagrangian: $\mathcal{L} = \left(\int_{i=0}^{1} C_i^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} + \lambda \left(S - \int_{i=0}^{1} P_i C_i di \right)$

- First-order conditions:

$$\frac{\partial \mathcal{L}}{\partial \lambda} = S - \int_{i=0}^{1} P_i C_i di = 0$$

$$\frac{\partial \mathcal{L}}{\partial C_i} = \frac{\eta}{\eta-1} \left(\int_{j=0}^{1} C_j^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}-1} \frac{\eta-1}{\eta} C_i^{\frac{\eta-1}{\eta}-1} - \lambda P_i = 0, \ \forall i \in [0,1]$$

Solving (1)

$$C_i^{-\frac{1}{\eta}} = \frac{\lambda}{\left(\int_{j=0}^{1} C_j^{\frac{\eta-1}{\eta}} dj\right)^{\frac{1}{\eta-1}}} P_i$$

$$C_i = \frac{\lambda^{-\eta}}{\left(\int_{j=0}^{1} C_j^{\frac{\eta-1}{\eta}} dj\right)^{-\frac{\eta}{\eta-1}}} P_i^{-\eta} = C\lambda^{-\eta} P_i^{-\eta} = AP_i^{-\eta}$$

- We don't know λ, so we don't know *A*.

Solving (2)

- Plug last expression for C_i into the definition of S:

$$\int_{j=0}^{1} P_j A P_j^{-\eta} dj = S$$

$$A \int_{j=0}^{1} P_j^{1-\eta} dj = S$$

$$A = \frac{S}{\int_{j=0}^{1} P_j^{1-\eta} dj}.$$

- Following steps in notes (and Romer) the denominator below is a natural price index P for this mode:

$$C = \frac{S}{\left(\int_{i=0}^{1} P_i^{1-\eta} di\right)^{\frac{1}{1-\eta}}} \equiv \frac{S}{P}.$$

Solution for C_i

- Putting all of this together, we get the demand equation for the *i*th good as

$$C_i = \left(\frac{P_i}{P} \right)^{-\eta} C$$

- Consumption of *i* depends on its relative price with elasticity η
- Any increase in total consumption *C* increases each good's consumption proportionally

Next steps

- We have the demand for each good
- In the next class, we will:
  - Examine labor supply of households, which can be analyzed separately because of the additive utility function
  - Examine the behavior of firms
  - Add in a simple representation of aggregate demand
  - Put it all together to characterize equilibrium in the model