New Keynesian IS/LM Model with Fixed Prices

- Suppose that we have extreme nominal rigidity: P is totally fixed over some relevant time horizon (over which other variables may vary)
- Can we derive the IS and LM curves from a model in which households maximize utility?
  - Sort of. The New Keynesian IS/LM model has much in common with traditional one.
  - It has more rigorous microfoundations, but has some unusual and perhaps unrealistic implications.
- $Y = F(L)$
  - No capital for simplicity
  - Positive and non-increasing marginal product of labor
  - Analogous to "short-run production function" in micro: capital is fixed factor and labor is variable
- $\mathrm{U} = \sum_{t=0}^{\infty} \beta^t \left[U(C_t) + \Gamma\left(\frac{M_t}{P_t}\right) - V(L_t) \right]$
  - Additive utility at t is function of components due to consumption, real money balances, and labor effort.
    - Additive form is easy to analyze because marginal utility of one component does not depend on the levels of the others
    - Might not be totally realistic: higher consumption probably raises the utility of money holding
  - This is short-cut, money-in-the-utility-function approach to why households demand money.
  - β is the discount factor, which is analogous to $e^{-\rho}$ or $\frac{1}{1+\rho}$
  - Usual assumptions about diminishing marginal utility of goods, money, and leisure lead to
    - $U' > 0, U'' < 0$
      - We will assume that $U(C_t) = \frac{C_t^{1-\theta}}{1-\theta}$
    - $\Gamma' > 0, \Gamma'' < 0$
      - We assume $\Gamma\left(\frac{M_t}{P_t}\right) = \frac{(M_t / P_t)^{1-\chi}}{1-\chi}$
    - $V' > 0, V'' > 0$
- $A_{t+1} = M_t + (A_t + W_t L_t - P_t C_t - M_t)(1 + i_t)$
  - A = nominal wealth

- Expression in parentheses is the amount of bonds household holds during period t, which is initial assets in t plus income minus consumption, minus the amount held as money.
- Utility maximization leads to Euler equation like that in Diamond model, except that β now plays the role of $1/(1 + \rho)$:
    - $MU(C_t) = (1+r_t)\beta \times MU(C_{t+1})$, or
    - $C_t^{-\theta} = (1+r_t)\beta C_{t+1}^{-\theta}$, or in log terms,
    - $\ln C_t = \ln C_{t+1} - \frac{1}{\theta}\ln\left[(1+r_t)\right] - \frac{1}{\theta}\ln(\beta)$
        - Note that we are using $1+r \equiv \frac{1+i}{1+\pi} \equiv \frac{1+i}{P_{t+1}/P_t}$
- In the aggregate, $C = Y$ (remember that there is no capital, so no investment), so the demand for consumption = the aggregate demand for output
    - Setting $a = -\frac{1}{\theta}\ln\beta$, which is a constant, substituting Y for C, and noting that for small r, $\ln(1+r) \approx r$, we get the

      New Keynesian IS curve: $\ln Y_t = a + \ln Y_{t+1} - \frac{1}{\theta} r_t$.
    - This looks like an IS curve because it is a negative relationship between r and Y, but it is really something quite different from the Keynesian version:
        - The key underlying relationship in the traditional IS curve is investment, which does not exist here. How can we explain cyclical variations, which are most pronounced in investment, without even having it in the model?
            - To the extent it is in there at all, the effect of r leads to an increase in saving (a rising desired consumption path), which would be *more* investment, not less.
        - The presence of future Y in this expression would mirror expected future income in the traditional consumption function, so that conforms
- Equilibrium in money holding
    - The budget constraint can be rewritten as

$$A_{t+1} = M_t + \left(A_t + W_t L_t - P_t C_t - M_t\right)(1+i_t)$$
$$= \left(A_t + W_t L_t - P_t C_t\right)(1+i_t) - i_t M_t$$
$$\frac{A_{t+1}}{P_t} = \left(\frac{A_t}{P_t} + \frac{W_t}{P_t}L_t - C_t\right)(1+i_t) - i_t \frac{M_t}{P_t}$$
$$\frac{A_{t+1}}{P_{t+1}}(1+\pi_{t+1}) = \left(\frac{A_t}{P_t} + \frac{W_t}{P_t}L_t - C_t\right)(1+i_t) - i_t \frac{M_t}{P_t}$$

  - This equation shows that the opportunity cost of holding an additional dollar of real money balances $\frac{M_t}{P_t}$ is $\frac{i_t}{(1+i_t)} \approx i_t$ units of consumption
  - The nominal interest rate is the opportunity cost of money holding: it is the forgone return that one could have obtained by holding wealth as bonds or capital instead of as money
- Following Romer's analysis, the household along its budget constraint trades off dm units of M/P for $\frac{i_t}{1+i_t}dm$ units of consumption at the same time t, so the marginal utility of these magnitudes must be equal for a household that is maximizing utility
  - Marginal utility of dm units of money is $\Gamma'(M_t / P_t)dm$
  - Marginal utility of $\frac{i_t}{1+i_t}dm$ units of consumption is $U'(C_t)\frac{i_t}{1+i_t}dm$
  - Balancing implies a first-order condition:

$$\Gamma'\left(\frac{M_t}{P_t}\right) = \frac{i_t}{1+i_t}U'(C_t)$$

$$\left(\frac{M_t}{P_t}\right)^{-\chi} = \frac{i_t}{1+i_t}(C_t)^{-\theta}$$

- Solving this for real money demand and noting that $C = Y$,

$\frac{M_t}{P_t} = Y_t^{\theta/\chi}\left(\frac{1+i_t}{i_t}\right)^{1/\chi}$, which is the **real money-demand equation** in this model.

- The **LM curve** is the set of combinations of Y and r that, for given P (fixed by assumption), M (set by monetary authority), and expected π, leads to equilibrium between money supply and money demand.
- From the FOC:

$$i \approx \frac{i_t}{1+i_t} = \left(\frac{M_t}{P_t}\right)^{-\chi}(Y_t)^{\theta}$$

$$\ln i = \theta \ln Y_t - \chi \ln(M_t / P_t)$$

- This is the LM equation and it slopes upward in i and Y.
  - Note that $i_t \approx r_t + \pi^e_{t+1}$, so we can write this approximately as

$$r_t = \left(\frac{M_t}{P_t}\right)^{-\chi}(Y_t)^{\theta} - \pi^e_{t+1}$$

  - This allows us to put the IS and LM curves on the same diagram with r on the vertical axis and Y on the horizontal.

        - The LM curve shifts down (point for point) with increases in expected inflation
        - Increases in the real money supply, whether through $M\uparrow$ or $P\downarrow$, move the LM curve down and to the right.
        - Note that the LM curve approaches a horizontal asymptote as $i \to 0$ from above (the right)
            - This is the "zero lower bound" or "liquidity trap" situation
            - The nominal interest rate cannot reach zero because no one would want to hold bonds
            - If expected inflation is negative (prices are expected to fall), then this happens at positive real interest rates, which may result in the IS curve intersecting the LM on its nearly horizontal region with $i \approx 0$.
            - Japan through much of the 1990s and the US currently are in or near this situation

IS/LM and aggregate demand

- The aggregate-demand curve describes the relationship between the aggregate price level *P* and the quantity of output demanded in the economy.
    - We discussed a simplistic representation of aggregate demand in the first week using the quantity theory: $Y = \frac{MV}{P}$ with money and velocity assumed to be exogenous
- The IS/LM model can be used to derive a somewhat more robust form of the AD curve
    - Suppose that $P\downarrow$ with everything else that is exogenous held constant
        - (This would be expectations about π and Y)
    - Unless we are at the zero-lower-bound, the LM curve shifts down and to the right (only to the right at ZLB with no effect on equilibrium)
    - This leads to an increase in Y as the economy moves down along the downward-sloping IS curve (unless at ZLB)
    - Thus, the AD curve under the IS/LM model slopes downward (vertical at ZLB) and depends on Y, π^e, and expected future Y.
        - Also on G and other exogenous determinants of spending if we relax Ricardian equivalence and other restrictive assumptions