Econ 314 Spring 2018

Math-oriented problems

1. A dollars are invested at the interest rate r, yielding B dollars after t years. Assuming continuous compounding, this may be expressed as $B = Ae^{rt}$

   a. Solve for A.
   b. Solve for r.
   c. At an interest rate of 5% (0.05), what is the present value of $1000 ten years from now? Assume continuous compounding.
   d. A dollars are invested at a fixed rate of interest r, compounded continuously. After ten years, your investment has grown to $1000 and after fifteen years to $2000. Find A, the amount initially invested, and the rate of interest r.

2. Suppose a firm's profit function, Π, as a function of output, y, is $\Pi(y) = -y^4 + 6y^2 - 5$, and its output, y, as a function of labor, L, is $y(L) = 5\lambda^{2/3}$.

   a. Use the chain rule to find $d\Pi / dL$ from the two functions.
   b. Find a direct expression for profit Π as a function of labor L.
   c. Check your answer to a by taking the derivative of the equation you derived in b.
   d. At what level of labor is profit maximized?

3. A production function F has constant returns to scale in two inputs K and L if $F(\lambda K, \lambda L) = \lambda F(K, L)$ for all $\lambda > 0$.

   a. Show whether (or under what conditions) the Cobb-Douglas production functions $Y = K^{\alpha}(AL)^{\beta}$ has constant returns to scale in K and L taking A as fixed.
   b. Show whether (or under what conditions) the same Cobb-Douglas has constant returns in K and A, taking L as fixed.
   c. Calculate the marginal product of capital (with A and L fixed) and show that it depends only on K/AL if the production function has constant returns to scale.

4. Consider the following three equations which together specify an *IS* curve:

$$Y = C + I + G$$
$$C = a + b(Y - T)$$
$$I = c - dr,$$

with income Y, interest rate r, consumption C, investment I, government purchases G, taxes T, and a, b, c, and d coefficients greater than zero.

   a. Find a single equation for the *IS* curve by solving the system of equations for Y.
   b. Y and r are endogenous variables. Is the equation you derived linear in the endogenous variables?

   c. Consider the equation: $M/P = kY - hr$. Find the *LM* curve by solving this equation for r. (k and h are positive coefficients; M is the money supply; P is the price level.)
   d. Y and r are both endogenous in the *IS*/*LM* model. Treating the *IS* and *LM* equations you derived in a and c as a two-equation system, solve for Y and r in terms of the other variables of the model. (Note: This means finding an expression for Y and an

expression for r that do not involve any endogenous variables. We are, temporarily, treating P as an exogenous variable.)

e. The equation you derived in d is the aggregate-demand curve, which expresses Y as a function of P. Is Y a linear function of P? For a given value of P (and given values for the other exogenous variables), how would Y change if G increased by one unit? How much would Y change if both G and T increased by one unit?

f. The aggregate-demand curve can be equally well expressed by solving for P as a function of Y. Find the AD curve in terms of P. Using this equation, how would P change if M increased by 10% (*i.e.*, were multiplied by 1.10), if Y and the exogenous variables other than M did not change?

6. Calculate the derivatives of the following functions:

a. $y = e^{(2x + 5)}$

b. $y = \left(x^3 + 5\right) log\left(x\right)$

c. $y = \dfrac{4x^{-2} + 5}{4x^2 + 7x}$