

The fundamental underlying equation of dynamic programming is the Bellman equation. We can write this equation as

$$V_i(0) = \lim_{\Delta t \rightarrow 0} V_i(\Delta t) \equiv \lim_{\Delta t \rightarrow 0} \left\{ \int_{t=0}^{\Delta t} u(t | \text{state at } 0 = i) dt + e^{-\rho \Delta t} E[V(\Delta t | \text{state at } 0 = i)] \right\},$$

Where $V_i(0)$ is expected lifetime utility from date (0) of someone who is currently in state i . (Remember that there are three states in the Shapiro-Stiglitz model: E, S, and U.)

Our utility function is $U = \int_0^{\infty} e^{-\rho t} u(t) dt$, where

$$u(t) = \begin{cases} w(t) - \bar{e}, & \text{if state is E,} \\ w(t), & \text{if state is S, and} \\ 0, & \text{if state is U.} \end{cases}$$

1. What is the intuition and interpretation of the $u(t)$ function?
2. For someone in state E at time 0, the probability of still being in state E t time units later is e^{-bt} and the probability of having moved into state U is $(1 - e^{-bt})$. Explain intuitively why the Bellman equation for state E at time 0 can be written:

$$V_E(0) = \lim_{\Delta t \rightarrow 0} \left\{ \int_{t=0}^{\Delta t} e^{-\rho t} \left[e^{-bt} (w - \bar{e}) + (1 - e^{-bt})(0) \right] dt + e^{-\rho \Delta t} \left[e^{-b\Delta t} V_E(\Delta t) + (1 - e^{-b\Delta t}) V_U(\Delta t) \right] \right\}.$$