

In the Calvo model, each firm has a fixed probability α of changing its price each period, independent of its prior price-changing history. In addition, the firms discount future profits with discount factor $\beta < 1$, so profits from t years in the future are valued only β^t times as much as current profits.

As in the general price-setting model, firms that are setting a new price at time t set it to be a weighted average of expected future optimal prices: $x_t = \sum_{s=0}^{\infty} \omega_s E_t(p_{t+s}^*)$, with $\sum_{s=0}^{\infty} \omega_s = 1$.

Romer argues that the ω_s weights are proportional to $\beta^s q_s$, where q_s is the probability that the firm's price is still x_t in period $t + s$.

1. Explain why $q_s = (1 - \alpha)^s$.
2. What is $\sum_{s=0}^{\infty} \beta^s (1 - \alpha)^s$, both intuitively and mathematically?
3. What given that summation, what must ω_s be?