Economics 314 Spring 2020

Daily Problem #13 February 19

In Romer's human-capital model of Chapter 4, the production function is

$$Y(t) = K(t)^{\alpha}\left[A(t)H(t)\right]^{1-\alpha}$$

with $H(t) \equiv L(t)G(E)$, where E is average education of the labor force and $L(t)$ is the number currently working. There are a total of $N(t)$ people in the economy at time t. We assume that A grows at exogenous rate g and that L and N grow (with given E) at exogenous rate n. $\dot{K}(t) = sY(t) - \delta K(t)$.

1. Show that for $k \equiv \frac{K}{AH} = \frac{K}{ALG(E)}$, the model converges (for given E) to a steady-state with

$$k^* = \left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}} \text{ and } y^* = \left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

2. Show that the level of per-capita income at time t on the steady-state growth path is

$$\left(\frac{Y(t)}{N(t)}\right)^{*} = y^* A(t)G(E)\left(\frac{L(t)}{N(t)}\right)^{*}.$$

3. How does an increase in the level of E affect the steady-state growth path of Y/N?