Economics 314 — Spring 2020

Daily Problem #6 — February 6

One of the conditions for the continuous-time lifetime utility-maximization problem is the consumption Euler equation:

$$\frac{\dot{C}(t)}{C(t)} = \frac{r(t) - \rho}{\theta}.$$

We demonstrated the following properties in the two-period model:

- If $r > \rho$, then the budget constraint is steeper than the indifference curve at $C_1 = C_2$ and the household will maximize utility with $C_2 > C_1$.
- If $r < \rho$, then the budget constraint is flatter than the indifference curve at $C_1 = C_2$ and the household will maximize utility with $C_2 < C_1$.
- If $r = \rho$, then the budget constraint is tangent to and has the same slope as the indifference curve at $C_1 = C_2$ and the household will maximize utility with $C_2 = C_1$.
- If $r \neq \rho$, then the magnitude of the household's optimal deviation from $C_1 = C_2$ will depend on the amount of curvature in the indifference curves, which is determined by $1/\theta$.

1. In the continuous-time setting, what conditions on the sign of $\frac{\dot{C}}{C}$ correspond to the discrete-time outcomes:

   a. $C_2 > C_1$?

   b. $C_2 < C_1$?

   c. $C_2 = C_1$?

2. Show that each of the four bullet properties of the discrete-time model are also implied by the Euler equation.