Econ 312

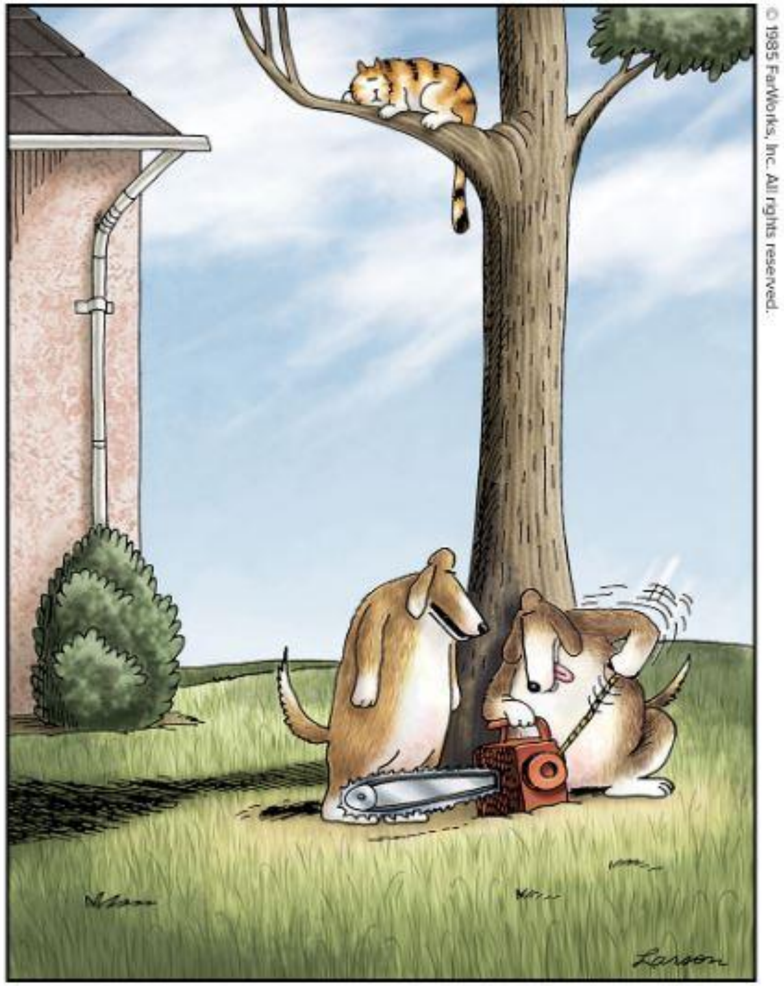
Monday, April 6

Vector Autoregression and Vector Error-Correction Models

Reading: Online time-series Chapter 5

Class notes: Pages 117 to 123

Today's Far Side offering



"You have to prime it, you know."

You may have noticed that the board outside my office tends to have a lot of dog comics, while Noel's has a lot of cat comics. I'm especially fond of comics in which dogs are asserting their obvious superiority to cats.

Context and overview

- We have talked a lot about estimating dynamic economic models with distributed lags of various kinds
  - All of these models have presumed that we were able to establish exogeneity
  - In many time-series applications (especially in macro) we cannot be confident that any of the variables is exogenous
- **Vector autoregression** was developed in the 1980s (Sims and others) to represent the **reduced-form** of a dynamic model that can be estimated by OLS without making strong assumptions about exogeneity

Simultaneous systems of equations

- We have not yet discussed estimation of **systems of simultaneous equations**, but we will later in the class
- In a simultaneous system, we have **multiple endogenous variables** and an equal number of equations
  - For example, price and quantity as endogenous variables and supply and demand curves as equations
- Such simultaneous systems cause trouble because there is usually an endogenous variable on the right-hand side
  - Price is endogenous in both the demand curve and the supply curve
- We'll study the **instrumental-variables** model for estimating the structural form of such models later

Reduced forms

- One solution: Solve out the endogenous variables to get the "**reduced form**"
  - Reduced-form system expresses each endogenous variable solely as a function of exogenous variables
  - Supply-demand system: One equation for price and one for quantity, with all exogenous variables that affect either supply or demand on the right-hand side of both reduced-form equations
- Because there are no endogenous variables on the right, the reduced-form equations can usually be estimated by OLS
- Neither reduced-form equation is "demand" or "supply;" both are combinations of both
- If the model is "**identified**," then we can reconstruct the demand and supply equations from the coefficients of the reduced form

Idea of vector autoregression (VAR)

- **Identification** is very problematic in dynamic macroeconomics
  - Nothing is really exogenous
  - Everything evolves together over time
- VAR models allow us to perform some tasks in dynamic models without identification
  - Forecasting
  - Granger "causality" tests
- Other tasks require us to make identification assumptions
  - Estimating the effects of a shock to one variable on others
  - Decomposing the variation in a variable into parts attributable to various shocks
- Variables in VARs should be stationary

A simple 2-variable model

- Following the time-series chapters and the daily problem, suppose we have a **structural model** of *y* and *x*

$$x_t = \alpha_0 + \alpha_1 x_{t-1} + \theta_0 y_t + \theta_1 y_{t-1} + \varepsilon_t^x$$

$$y_t = \phi_0 + \phi_1 y_{t-1} + \delta_0 x_t + \delta_1 x_{t-1} + \varepsilon_t^y$$

- Both *y* and *x* depend on current and lagged values of both variables
- Epsilon terms are the "**shocks**" to *x* and *y*
  - We assume that they are **white noise**

$$\operatorname{var}\left(\varepsilon_t^x\right) = \sigma_x^2,\ \operatorname{var}\left(\varepsilon_t^y\right) = \sigma_y^2,\ \operatorname{cov}\left(\varepsilon_t^x, \varepsilon_t^y\right) = \sigma_{xy}$$

Reduced-form solution

- The exogenous variables are just lagged *x* and *y* and the two shocks
- The reduced form expresses current *x* and *y* as functions solely of these exogenous variables
- Solving:

$$x_t = \frac{\alpha_0 + \theta_0\phi_0}{1-\theta_0\delta_0} + \frac{\alpha_1 + \theta_0\delta_1}{1-\theta_0\delta_0} x_{t-1} + \frac{\theta_1 + \theta_0\phi_1}{1-\theta_0\delta_0} y_{t-1} + \frac{\theta_0\varepsilon_t^y + \varepsilon_t^x}{1-\theta_0\delta_0}$$

$$y_t = \frac{\phi_0 + \theta_0\alpha_0}{1-\theta_0\delta_0} + \frac{\phi_1 + \delta_0\theta_1}{1-\theta_0\delta_0} x_{t-1} + \frac{\delta_1 + \delta_0\alpha_1}{1-\theta_0\delta_0} y_{t-1} + \frac{\delta_0\varepsilon_t^x + \varepsilon_t^y}{1-\theta_0\delta_0}.$$

  - BOTH equations contain parameters from BOTH structural equations
  - The "error terms" of BOTH equations contain BOTH shocks

Reduced form as VAR

- We give new names to the complicated coefficients in the reduced form:

$$x_t = \frac{\alpha_0 + \theta_0\phi_0}{1-\theta_0\delta_0} + \frac{\alpha_1 + \theta_0\delta_1}{1-\theta_0\delta_0} x_{t-1} + \frac{\theta_1 + \theta_0\phi_1}{1-\theta_0\delta_0} y_{t-1} + \frac{\theta_0\varepsilon_t^y + \varepsilon_t^x}{1-\theta_0\delta_0}$$

$$y_t = \frac{\phi_0 + \theta_0\alpha_0}{1-\theta_0\delta_0} + \frac{\phi_1 + \delta_0\theta_1}{1-\theta_0\delta_0} x_{t-1} + \frac{\delta_1 + \delta_0\alpha_1}{1-\theta_0\delta_0} y_{t-1} + \frac{\delta_0\varepsilon_t^x + \varepsilon_t^y}{1-\theta_0\delta_0}$$

$$x_t = \beta_{x,0} + \beta_{x,1} x_{t-1} + \gamma_{x,1} y_{t-1} + v_t^x$$

$$y_t = \beta_{y,0} + \beta_{y,1} x_{t-1} + \gamma_{y,1} y_{t-1} + v_t^y$$

Error terms in VAR equations

- Error terms in the VAR equations contain BOTH shocks:

$$v_t^x = \frac{\theta_0 \varepsilon_t^y + \varepsilon_t^x}{1 - \theta_0 \delta_0}, \quad v_t^y = \frac{\delta_0 \varepsilon_t^x + \varepsilon_t^y}{1 - \theta_0 \delta_0}$$

- Properties:

$$\operatorname{var}\left(v_t^x\right) = \frac{1}{\left(1 - \theta_0 \delta_0\right)^2} \left(\sigma_x^2 + \theta_0^2 \sigma_y^2 + 2\theta_0 \sigma_{xy}\right)$$

$$\operatorname{var}\left(v_t^y\right) = \frac{1}{\left(1 - \theta_0 \delta_0\right)^2} \left(\delta_0^2 \sigma_x^2 + \sigma_y^2 + 2\delta_0 \sigma_{xy}\right)$$

$$\operatorname{cov}\left(v_t^x, v_t^y\right) = \frac{2}{\left(1 - \theta_0 \delta_0\right)^2} \left(\delta_0 \sigma_x^2 + \theta_0 \sigma_y^2 + \left(1 + \delta_0 \theta_0\right) \sigma_{xy}\right)$$

Estimating the VAR

- We can estimate the VAR as a set of two **OLS** equations

$$x_t = \beta_{x,0} + \beta_{x,1} x_{t-1} + \gamma_{x,1} y_{t-1} + v_t^x$$

$$y_t = \beta_{y,0} + \beta_{y,1} x_{t-1} + \gamma_{y,1} y_{t-1} + v_t^y$$

- There can be $m > 2$ variables
  - But each variable adds regressors to all equations
- There can be $p > 1$ lags
  - Use AIC or Schwartz/Bayesian IC to figure out how many lags are required
- Each VAR equation has $m \times p$ variables on the right
  - $m \times p$ gets big quickly!

Using estimated VAR without identification

- **Forecast** y and x by running the VAR equations forward assuming zero (the expected value) for the error terms v

- Attempt to infer **causality** using a controversial (but common) technique developed by Granger

Granger causality

$$x_t = \beta_{x,0} + \beta_{x,1} x_{t-1} + \gamma_{x,1} y_{t-1} + v_t^x$$

$$y_t = \beta_{y,0} + \beta_{y,1} x_{t-1} + \gamma_{y,1} y_{t-1} + v_t^y$$

- Suppose that we fail to reject $H_0 : \gamma_{x,1} = 0$
- Lagged y does not help us predict x given the past behavior of x
  - This means that we cannot conclude that y "**Granger causes**" x
- If we fail to reject $H_0 : \beta_{y,1} = 0$ then lagged x does not help us predict y and we cannot conclude that x Granger causes y
- Any of four outcomes is possible: Neither may Granger cause the other, both may Granger cause each other, or Granger causality can run in either single direction

Is Granger causality really causality?

- In general, correlation does not imply causality
- Granger causality is only plausible if we assume
  - Causality is **NEVER purely instantaneous** (just period t causing period t)
  - That the **present cannot cause the past**
  - That there is **no third variable missing** from the system that causes both
- Because of the last assumption, you can get **different results** between x and y if you add z to the system
- Questionable restrictions, but Granger causality has been used a lot

Identification of VARs

- Identification assumptions may allow us to recover the parameters of the structural system from the reduced-form system
- We estimate

$$x_t = \beta_{x,0} + \beta_{x,1} x_{t-1} + \gamma_{x,1} y_{t-1} + v_t^x$$

$$y_t = \beta_{y,0} + \beta_{y,1} x_{t-1} + \gamma_{y,1} y_{t-1} + v_t^y$$

- Use estimates of β and γ parameters to infer parameters of

$$x_t = \alpha_0 + \alpha_1 x_{t-1} + \theta_0 y_t + \theta_1 y_{t-1} + \varepsilon_t^x$$

$$y_t = \phi_0 + \phi_1 y_{t-1} + \delta_0 x_t + \delta_1 x_{t-1} + \varepsilon_t^y$$

Are there enough parameters?

- Reduced form has 6 coefficients, 2 error variances, and 1 covariance = **9 pieces of information**

$$x_t = \beta_{x,0} + \beta_{x,1} x_{t-1} + \gamma_{x,1} y_{t-1} + v_t^x$$

$$y_t = \beta_{y,0} + \beta_{y,1} x_{t-1} + \gamma_{y,1} y_{t-1} + v_t^y$$

- Structural form has 8 coefficients, 2 error variances, and 1 covariance = **11 things we want to know**

$$x_t = \alpha_0 + \alpha_1 x_{t-1} + \theta_0 y \ + \theta_1 y_{t-1} + \varepsilon_t^x$$

$$y_t = \phi_0 + \phi_1 y_{t-1} + \delta_0 x_t + \delta_1 x_{t-1} + \varepsilon_t^y$$

- We need 11 – 9 = **2 additional restrictions** to identify model

Possible identification restrictions

$$x_t = \alpha_0 + \alpha_1 x_{t-1} + \theta_0 y_t + \theta_1 y_{t-1} + \varepsilon_t^x$$
$$y_t = \phi_0 + \phi_1 y_{t-1} + \delta_0 x_t + \delta_1 x_{t-1} + \varepsilon_t^y$$

1. Nearly always assume **independence of structural shocks**, $\sigma_{xy} = 0$
2. Most common second assumption is either $\theta_0 = 0$ or $\delta_0 = 0$
   - If $\theta_0 = 0$, then y affects x only with a lag
   - If $\delta_0 = 0$, then x affects y only with a lag
   - This allows us to interpret the **contemporaneous correlation** between x and y by making an **assumption about instantaneous causality**

- Other assumptions may be used in place of 2: "structural VARs" or "identified VARs"

With more than two variables

- With m variables and equations, you need **$m(m - 1)/2$ assumptions** in addition to independence of shocks
- Most common assumption is "**ordering**" of contemporaneous causality
  - Assume an order of the variables w, x, y, z such that variables later on the list have no contemporaneous effect on those before them
  - Here, w can affect all variables immediately; x only affects y and z right away and affects w only with lag (#1); y only affects z now and affects w and x with lag (#2, 3); and z has only lagged effects on the others (#4, 5, 6)
  - This is $4(4 - 1)/2 = 6$ assumptions, as required

Impulse-response functions

- **Impulse-response function**: Most common use of identified VARs
- IRF tells us how a structural shock to either variable affects both variables:

$$\frac{\Delta E\left(x_{t+s}\right)}{\Delta \varepsilon_t^x} \text{ and } \frac{\Delta E\left(y_{t+s}\right)}{\Delta \varepsilon_t^x} \text{ for } s = 0, 1, \ldots$$

$$\frac{\Delta E\left(x_{t+s}\right)}{\Delta \varepsilon_t^y} \text{ and } \frac{\Delta E\left(y_{t+s}\right)}{\Delta \varepsilon_t^y} \text{ for } s = 0, 1, \ldots$$

- Two-variable system means 2×2 IRF sequences
- Usually presented as set of graphs with s on the horizontal axis

IRFs only possible in identified VARs

- You **must identify** your VAR with appropriate assumption(s) to calculate impulse-response functions
  - In general, both reduced-form v error terms are non-zero in each period
  - These cannot be interpreted as shocks to either x or y: combinations of both
  - Must know how to interpret contemporaneous correlation in order to distinguish structural shocks to x from structural shocks to y
- Different identifying assumptions may lead to different results

Example of IRFs from Romer & Romer (2010)

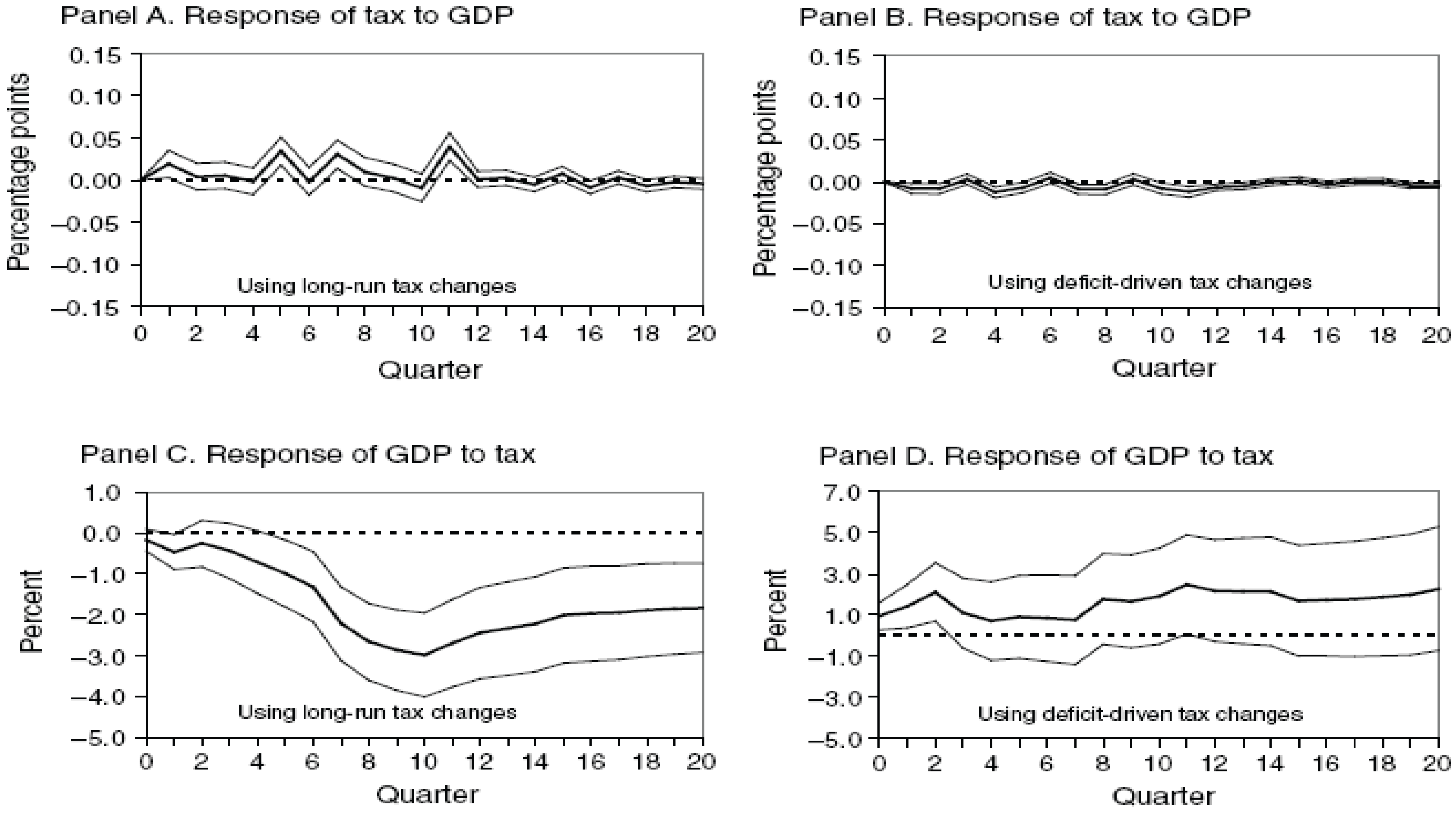


Figure 9. Results of Two-Variable VARs for the Two Types of Exogenous Tax Changes and GDP

Variance decompositions

- Another, less-common use of identified VARS
- Answers the question "How much of the variance in y_{t+s} is due to shocks to x_t vs. shocks to y_t?"
- Formulas can be found in time-series textbooks

VARs in Stata

- var varlist , lags(1/4) does estimation with lags 1 through 4
  - Can also include exog(vars) to add variables but not equations
- After var command:
  - fcast compute (then fcast graph)
  - oirf (create, graph, table)
    - oirf does ordered identification (specific order explicitly or list in desired order)
    - irf alone assumes neither has immediate effect
  - vargranger
  - varlmar (test for autocorrelated residuals)
  - varsoc (criteria for lag length)
  - varstable (check stability of model)
- var svar does structural VARs

Vector error-correction (VEC) models

- Can do VAR systems with cointegrated variables using **vector error-correction**
- There can be as many as $m - 1$ cointegrating relationships among m variables
- VEC model is:
  - **VAR in the differences** of the variables (which are stationary)
  - **Adding error-correction terms** corresponding to any or all cointegrating relationships (which are also stationary)
- Stata `vec` command will estimate and analyze VEC systems

Review and summary

- **Vector autoregression** is a method of estimating the dynamic relationship among a set of variables
- Without any **identifying assumptions**, we can use VARs for **forecasting** and testing **Granger causality**
- With appropriate identifying assumptions, we can estimate **impulse-response functions** to see how shocks to any variable affects others, and we can use **variance decompositions** to estimate which kinds of shocks are most important to the movement of each variable
- With cointegrated variables, we use the **vector error-correction** model

Something different: A puzzle

Given that this is a quantitative class, a numerical puzzle seems appropriate:

What is the pattern in the following numerical sequence?

8, 5, 4, 9, 1, 7, 6, 10, 3, 2

[Using the Internet to find the solution is cheating!]

What’s next?

- This class ends our segment on time-series analysis
  - (Whew!)
- In the next section we start looking at models for **pooled and panel-data** samples: those that combine a time-series dimension with cross sections
  - In the first class (April 8) we will consider pooled samples and the **fixed-effects estimator** for panel samples
  - In the second (April 10) we will introduce the **random-effects estimator** and do a detailed example together