Econ 312

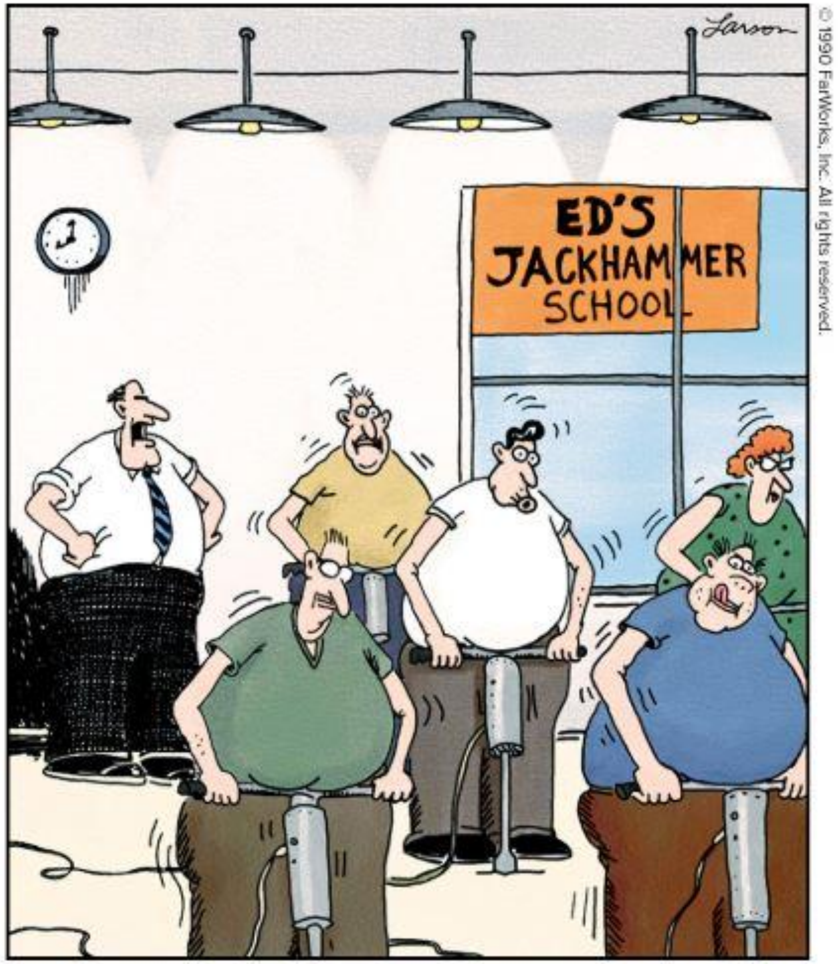
Friday, April 3

Regression with Integrated Variables:
Testing for Unit Roots and Cointegration Models

Reading: Online time-series Chapter 4

Class notes: Pages 111 to 117

Today’s Far Side offering



“C’mon! Keep those stomachs over the handles! Let the fat do the work! Let the fat do the work! ... That’s it!”

Finally! An occupation well-suited to my body type! ☺

Context and overview

- **Last class**: We examined distributed-lag regression models for estimating dynamic relationships among stationary variables
- **Today**: This class introduces regressions for integrated (difference stationary) variables, including
  - Testing for **unit roots**
  - **Spurious regressions** with integrated variables
  - **Cointegration** among variables
  - **Error-correction** models for cointegrated variables

Unit-root tests for non-stationarity

- **Dickey-Fuller** (DF) test is most basic
- **Augmented Dickey-Fuller** (ADF) adds lags seeking dynamic completeness
- **Phillips-Perron** test is Dickey-Fuller with HAC robust standard errors rather than lags
- **DF-GLS** test is alternative proposed by Stock and Watson to use GLS quasi-differencing to improve low power of DF and related tests

Dickey-Fuller test for random walk

- Consider AR(1) process: $y_t = \rho y_{t-1} + u_t$
- Null hypothesis is that it is a random walk
  - H_0: $\rho = 1$, H_1: $\rho < 1$
  - Under null, y is $I(1)$; under one-tailed alternative, y is $I(0)$
  - We can't just use OLS t statistic because under null hypothesis y is $I(1)$, which violates TS assumptions
- Subtract lagged y from both sides to get

$$\Delta y_t = (\rho - 1) y_{t-1} + u_t = \gamma y_{t-1} + u_t, \quad \gamma = \rho - 1$$

- Null hypothesis is now $\gamma = 0$ (non-stationarity) vs. $\gamma < 0$ (stationarity)

Dickey-Fuller test statistic

- We estimate $\hat{\gamma}$ by OLS from $\Delta y_t = \gamma y_{t-1} + u_t$, and calculate the "*t*" statistic by dividing it by its standard error
  - Note there is no constant term in the Dickey-Fuller regression
- This **DOES NOT follow a *t* distribution** because the regressor is $I(1)$ under H_0
- Dickey and Fuller used Monte Carlo methods to compute critical values for the one-tailed test of $\gamma = 0$ vs. $\gamma < 0$
  - Table 18.2 on page 611 of Wooldridge
  - Values are much larger (in absolute value) than the 1.96 we often use
- Reject the presence of a unit root if test statistic is more negative than the critical value

Logic of Dickey-Fuller test: mean-reversion

- Basic estimating equation is $\Delta y_t = \gamma y_{t-1} + u_t$
- We test whether γ is negative: if we reject null, then y is $I(0)$
- If γ < 0, then a **high value of *y* last period → decrease in *y* this period**
  - This means it is **reverting back toward a fixed mean** (zero in this case)
  - That is a basic property of **stationary variables**
- If we can be statistically confident that γ < 0 (*i.e.*, reject the null of γ = 0), then we conclude that *y* is a stationary, mean-reverting variable
- If we cannot reject γ = 0, then ρ might be 1 and *y* might be $I(1)$

DF tests for random walk with drift

- Many variables tend to grow over time; these can be **random walks with drift**
  - For this, we add a constant term to the Dickey-Fuller regression

$$y_t = \alpha + \rho y_{t-1} + u_t$$

$$\Delta y_t = \alpha + (\rho - 1) y_{t-1} + u_t = \alpha + \gamma y_{t-1} + u_t$$

$$H_0 : \rho = 1 \; (\gamma = 0)$$

$$H_1 : \rho < 1 \; (\gamma < 0)$$

- Under null, $\Delta y_t = \alpha + u_t$ and y is a random walk with constant drift α
- Very similar to basic DF test, but different critical values (Table 18.3 on p. 613 of Wooldridge)

Autocorrelated error in DF regression

- What if the error term u in the Dickey-Fuller regression is autocorrelated?
  - This is common in all time-series regressions
  - The tabulated DF test statistics assume that u is white noise
- Two choices
  1. **Augmented Dickey-Fuller (ADF)** test adds $p > 0$ lags of the dependent variable Δy to the right-hand side to make model dynamically complete
     - Critical values depend on p
  2. **Phillips-Perron test** doesn't add lags, but uses Newey-West HAC robust standard errors to calculate test statistic

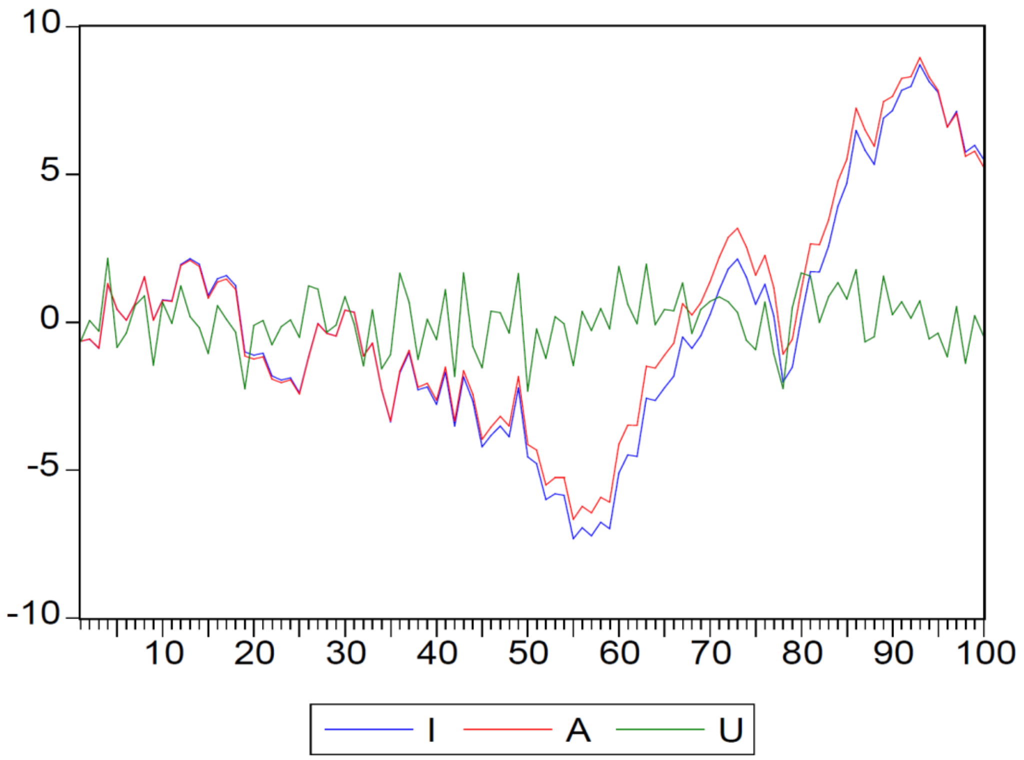
Testing for unit roots in Stata

- Dickey-Fuller and ADF tests: **dfuller** command
  - Options
    - **noconstant** suppresses constant to test random walk without drift
    - **drift** adds a constant to test random walk with drift
    - **trend** adds a trend to test for trend-stationary series
    - **lags(#)** adds # lags to use ADF rather than DF test
- Phillips-Perron test: **pperron** command
  - Options
    - **noconstant** and **trend** have same meaning here (drift is default)
    - **lags(#)** is the number of lags in the Newey-West approximation, not lags of Δy

The problem with low power

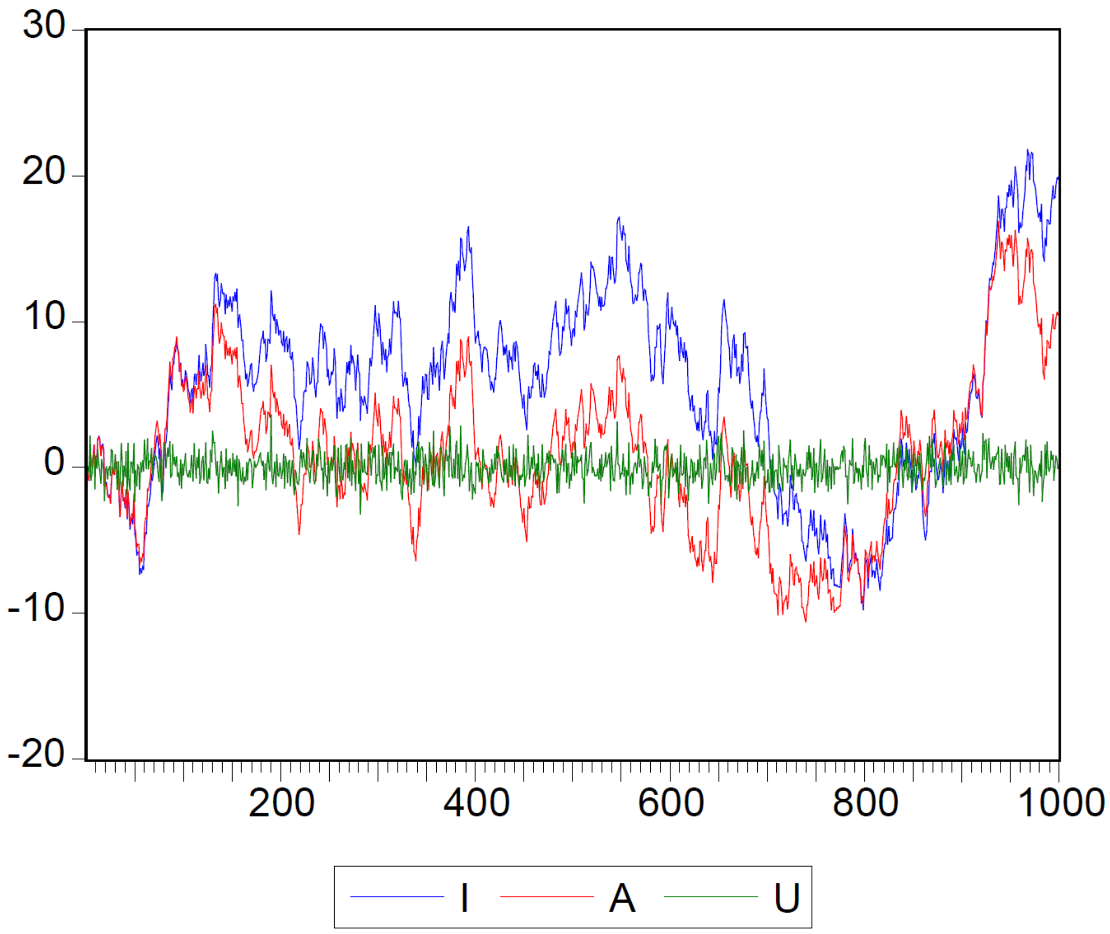
- Dickey-Fuller and Phillips-Perron tests tend to have **"low power"**
  - Often fail to reject false null hypotheses
  - Often can't prove stationarity (alternative hypothesis) even when it is true (and null is false)
  - If we decide to conclude non-stationarity whenever we fail to reject these tests, we will mistake a lot of stationary series for non-stationary
- Problem is borderline, but stationary processes
  - Random walk is nonstationary $y_t = y_{t-1} + u_t$
  - Stationary process $y_t = 0.9999 y_{t-1} + u_t$ is almost identical
  - Is it possible to distinguish between them?

100 observations



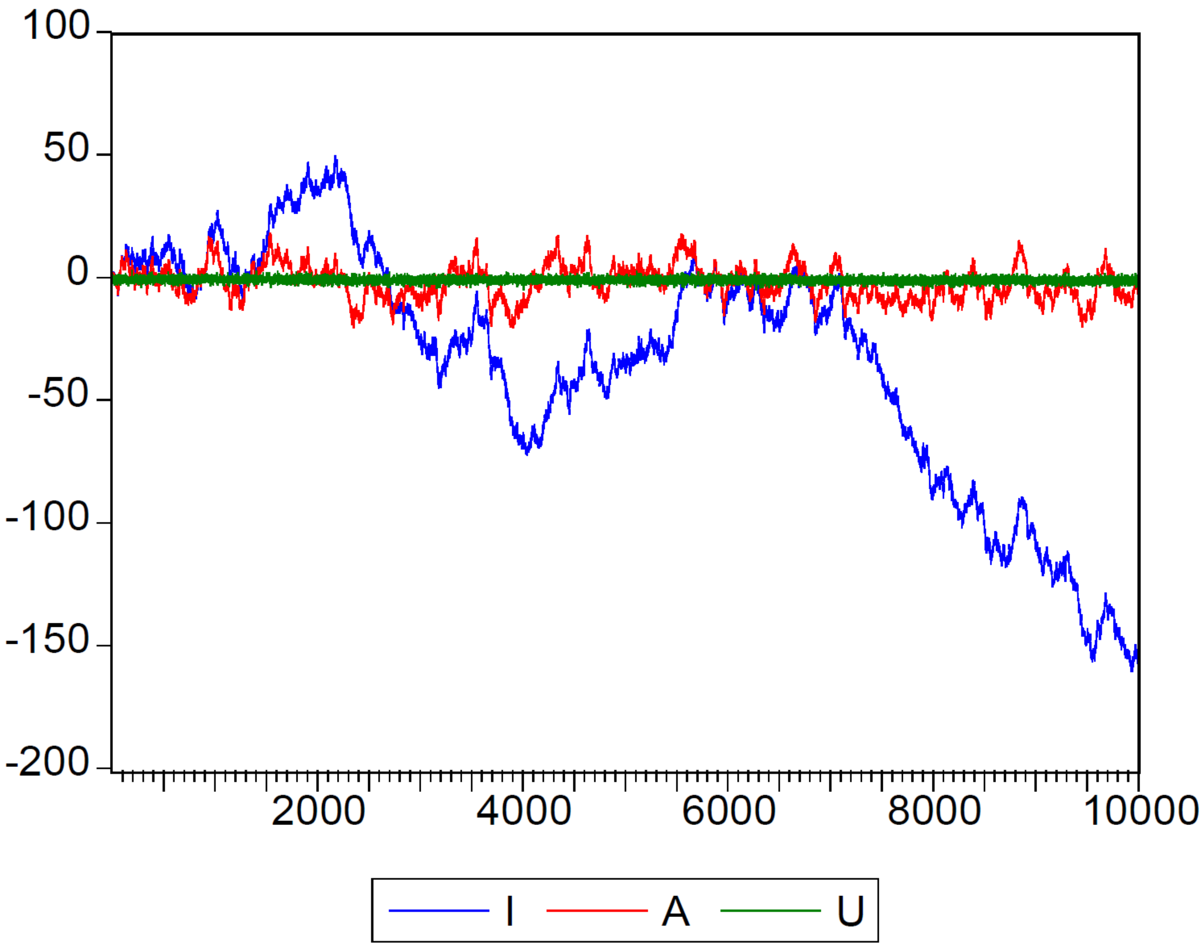
- Blue series I is integrated random walk (nonstationary)
- Red series A is AR(1) with coefficient 0.9999 (stationary)
- Green series U is underlying white noise process
- With $T = 100$, it's very hard to tell I from A

1,000 observations



- With $T = 1000$, the stationary autoregressive process (red) still looks a lot more like the random walk (blue)
- Seeing them together one can tell which one is mean-reverting
- Could you tell if I just showed you one?

10,000 observations



- With $T = 10000$, we can finally distinguish easily between the just-barely-stationary red series and the non-stationary blue
- Do we usually have 10000 observations? Do we EVER have 10000 observations?
- This is the reason why unit-root tests have low power

Another test that may have more power

- **DF-GLS** test was developed by Stock and Watson
- They claim that they get **more power** by quasi-differencing the series before running a DF-style test
- Details are on pages 113 and 114 of class notes
- **dfgls** command performs this test in Stata

Cointegration

- Normally, we must take the difference of $I(1)$ variables before using them in regressions: avoid spurious regressions
- Special case of **cointegration**:
  - Two or more variables that follow a **COMMON stochastic trend**
  - Each variable moves in a nonstationary way, like a random walk
  - There is some stationary, long-run relationship that ties the variables together
  - "Two variables taking a random walk together"
- This is important in economics:
  - Stable long-run relationships are common among $I(1)$ variables

Integration without cointegration

- Recall Granger and Newbold's spurious regression problem
- Suppose both y and x are $I(1)$ and they are not cointegrated:
- Equation in levels: $y_t = \beta_0 + \beta_1 x_t + u_t$
  - This would be spurious regression if estimated in levels
- Equation in differences: $\Delta y_t = \beta_1 \Delta x_t + \Delta u_t$.
  - Both differences and the error term are $I(0)$, so no problem estimating with OLS
  - Note absence of constant term, which "differences away"
  - Including a constant in differenced equation = including time trend in levels

Are bygones bygones: Is *u* stationary?

- If $u_t = y_t - \beta_0 - \beta_1 x_t$ is ***I*** **(1)**, then there is **no tendency for it to revert to zero**
  - No long-term, stable relationship between levels of *y* and *x*
  - Large error (disequilibrium) in period *t* would not be corrected in *t* + 1
  - Bygones are bygones: Changes in Δy_{t+1} does NOT depend on what happened in *t* or before
  - Differenced equation is best way to estimate
- If $u_t = y_t - \beta_0 - \beta_1 x_t$ is ***I*** **(0)**, then it **reverts to zero**
  - Deviations from $y_t - \beta_0 - \beta_1 x_t$ go away and *y* reverts to $\beta_0 + \beta_1 x_t$
  - If $y_t > \beta_0 + \beta_1 x_t$ due to positive shock in *t*, then Δy_{t+1} will **tend to be negative** to bring *y* back into its long-run equilibrium relationship with *x*
  - Estimating in differenced form **loses this long-run relationship**
  - This is the **cointegration** model and requires a different estimator

Error-correction models for cointegration

- Long-run equilibrium equation ("cointegrating regression"):

  $$y_t = \beta_0 + \beta_1 x_t + u_t \quad \text{with } u \sim I(0)$$

- Short-run adjustment equation ("error-correction model")

  $$\Delta y_t = -\alpha\left(y_{t-1} - \beta_0 - \beta_1 x_{t-1}\right) + \theta_1 \Delta y_{t-1} + \ldots + \theta_p \Delta y_{t-p} + \delta_0 \Delta x_t + \ldots + \delta_q \Delta x_{t-q} + v_t$$

  - This equation describes the short-run dynamics of *y* and its convergence back to long-run equilibrium with *x*
  - The term in parentheses in ECM is u_{t-1}
  - $\alpha > 0$, so if *y* was above equilibrium in $t - 1$, then Δy tends to be negative in *t*
  - The lagged Δy and Δx terms are rational lag to make model dynamically complete, so *v* is white noise
  - All terms in ECM are $I(0)$, so it can be estimated by OLS

Estimating an error-correction model

- Could use **nonlinear LS** to estimate all parameters of ECM together
- Simpler: Estimate cointegrating regression (CR) first, then ECM
  1. **CR by OLS**: estimates are "super-consistent"
     - Can't use *t* statistics to test hypothesis due to spurious regression concerns
     - But we get excellent estimates of the parameters
  2. **ECM by OLS imposing estimated β** (cointegrating vector) from CR
     - We would usually need to take account of the fact that the lagged cointegration term involves estimated parameters
     - Not in this case because they are super-consistent
- Multi-variate cointegration?
  - Sure!
  - If we have m variables that are $I(1)$, there can be up to $m-1$ cointegrating relationships among them reflecting long-run equilibrium relationships

Testing for cointegration

- **Engle-Granger** test
  - Estimate potential CR in levels and test residuals for unit root using ADF test
  - Critical values will be different than standard ADF test because these are residuals rather than a variable itself

- **Johansen-Juselius** test
  - More complicated, but generalizes easily to testing for more than one cointegrating relationship among more than two variables

Review and summary

- We can **test** a single variable for stationarity using Dickey-Fuller, augmented Dickey-Fuller, or Phillip-Perron tests
  - These tests tend to have low power to discriminate between non-stationary and barely-stationary variables
- Sets of non-stationary variables are **cointegrated** if there is a stable (stationary) long-run relationship among them
- Relationships among cointegrated variables can be estimated by **error-correction models**

From *The Devil's Dictionary*

Riot, *n*. A popular entertainment given to the military by innocent bystanders.

What's next?

- The next class covers **vector autoregression (VAR)**, a flexible technique for estimating dynamic relationships among a group of variables

- VARs are the go-to method for most time-series analysis in macroeconomics