

Economics 312  
Daily Problem #32

Spring 2020  
April 10

Consider the panel-data regression model  $y_{it} = \beta_{0i} + \beta_1 x_{1it} + \beta_2 x_{2it} + u_{it}$ . Suppose that we consider the individual-specific intercept coefficient to reflect random variation across individuals around a fixed mean:  $\beta_{0i} = \bar{\beta}_0 + a_i$ ,  $E(a_i) = 0$ ,  $\text{cov}(a_i, a_j) = 0$ ,  $i \neq j$ ,  $\text{var}(a_i) = \sigma_a^2$ .

1. Show that this “random-effects” model can be written as  $y_{it} = \bar{\beta}_0 + \beta_1 x_{1it} + \beta_2 x_{2it} + v_{it}$ , where  $v_{it} = a_i + u_{it}$ .

2. Assuming that  $E(u_{it}) = 0$ ,  $\text{var}(u_{it}) = \sigma_u^2$ , and  $\text{cov}(u_{it}, a_i) = 0$ , find

(a)  $E(v_{it})$ ,

(b)  $\text{var}(v_{it})$ ,

(c)  $\text{cov}(v_{it}, v_{is})$ ,  $s \neq t$ ,

(d)  $\text{cov}(v_{it}, v_{js})$ ,  $i \neq j, s \neq t$ .

3. Explain intuitively how we could attempt to estimate  $\sigma_u^2$  and  $\sigma_a^2$  using OLS residuals  $\hat{v}_{it}$ . Given the properties of  $v_{ij}$ , would you expect these estimates to be consistent? (Hint: They will be consistent if the OLS coefficient estimators are consistent.)

4. If we had estimates of  $\sigma_u^2$  and  $\sigma_a^2$ , explain in principle how we could use GLS to get asymptotically efficient estimators of the coefficients.