Economics 312 — Spring 2020

Daily Problem #23 — March 12

One of the most common time-series processes is the first-order autoregressive process: AR(1). Suppose that the error term of a time-series regression follows an AR(1) process: $(1-\rho L)u_t = \varepsilon_t$, or $u_t = \rho u_{t-1} + \varepsilon_t$, where $-1 < \rho < 1$ and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ is serially uncorrelated white noise.

1. Using the result of Daily Problem #22, show that we can write $u_t = \sum_{s=0}^{\infty} \rho^s \varepsilon_{t-s}$.

2. Using the property that $\text{cov}(\varepsilon_t, \varepsilon_{t-s}) = 0$ for all $s \neq 0$, use the expression in question 1 to show that $\text{var}(u_t) \equiv \sigma_u^2 = \sigma_\varepsilon^2 \sum_{s=0}^{\infty} (\rho^2)^s = \frac{\sigma_\varepsilon^2}{1-\rho^2}$. (Hint: Use the equation $u_t = \rho u_{t-1} + \varepsilon_t$, take the variance of both sides, and note that $\text{var}(u_t) = \sigma_u^2$ for all t and that ε_t is uncorrelated with anything that happened before t.)