Economics 312 — Spring 2020
Daily Problem #22 — March 11

The lag operator L is a time-series operator that takes a time-series variable and moves it back one period: $L(x_t) \equiv x_{t-1}$. When working with a polynomial expression involving different lags of the same variable, it is common to write the expression as a polynomial in the lag operator "times" the variable. For example, we could write $x_t - 0.5x_{t-1} + 0.1x_{t-2}$ as $(1 - 0.5L + 0.1L^2)x_t$.

The problems below are designed to help accustom you to working with the lag operator.

1. Write the following expressions in terms of the lags of x:
    a. $(1 - \alpha L)x_t$
    b. $(1 - \alpha L^2)x_t$
    c. $(1 - \alpha L)^2 x_t$
    d. $(1 - L)x_t$

2. Show that if $y_t - \alpha y_{t-1} = (1 - \alpha L)y_t = x_t$, then $y_t = (1 - \alpha L)^{-1} x_t = x_t + \alpha x_{t-1} + \alpha^2 x_{t-2} + \ldots = \sum_{s=0}^{\infty} \alpha^s x_{t-s}$.
For what values of the parameter α does the effect of x_{t-s} on y_t dissipate as s gets large?

3. As a special case of this, show that if $y_t - y_{t-1} = (1 - L)y_t = x_t$, then $y_t = \sum_{s=0}^{\infty} x_{t-s}$. What happens to the effect of x_{t-s} on y_t as s gets large in this case?