Economics 312 — Spring 2020
Daily Problem #21 — March 9

A textbook data set (from Hill, Griffiths, and Lim) vacation.dta gives us data on the vacation travel of 200 Chicago households. The variable definition file gives the following definitions and summary statistics:

```
       1. miles     miles traveled per year
       2. income    annual income in $1000's
       3. age       average age of adult members of household
       4. kids      number of children in household
```

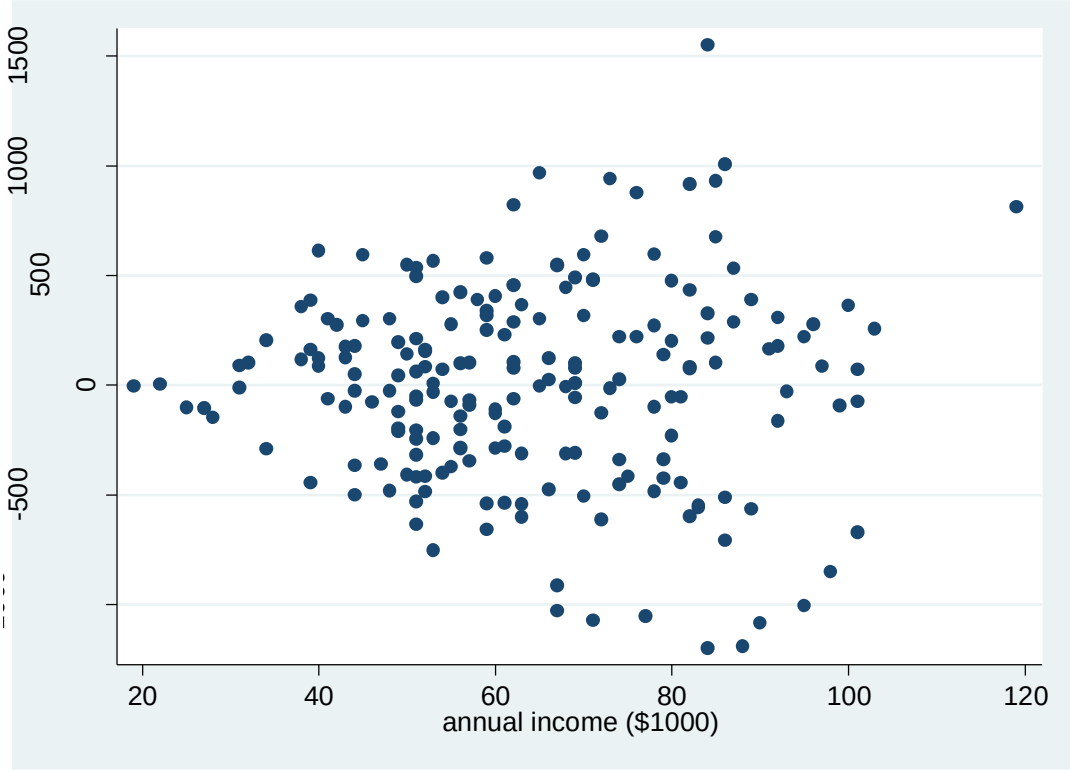
```
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
       miles |       200     1054.23     552.799          0       2609
      income |       200      63.925    18.33412         19        119
         age |       200      42.675    9.349139         23         59
        kids |       200       1.635    1.260842          0          4
```

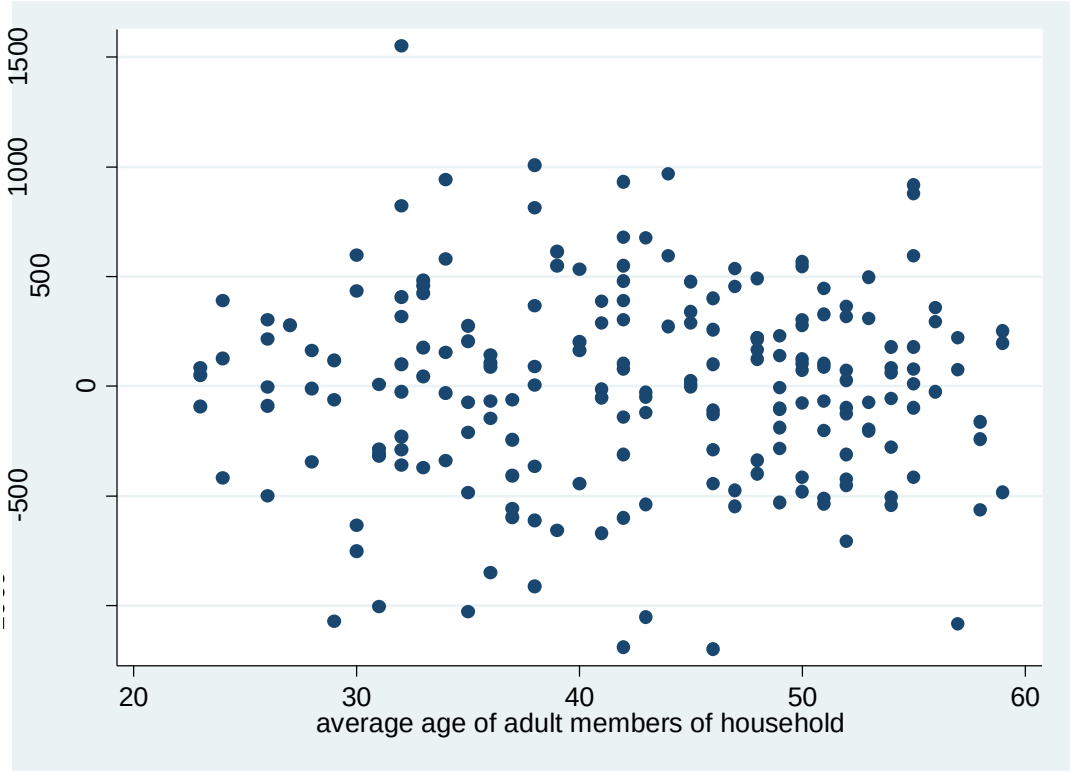
A regression of the model $miles = \beta_1 + \beta_2 income + \beta_3 age + \beta_4 kids$ yields the following:

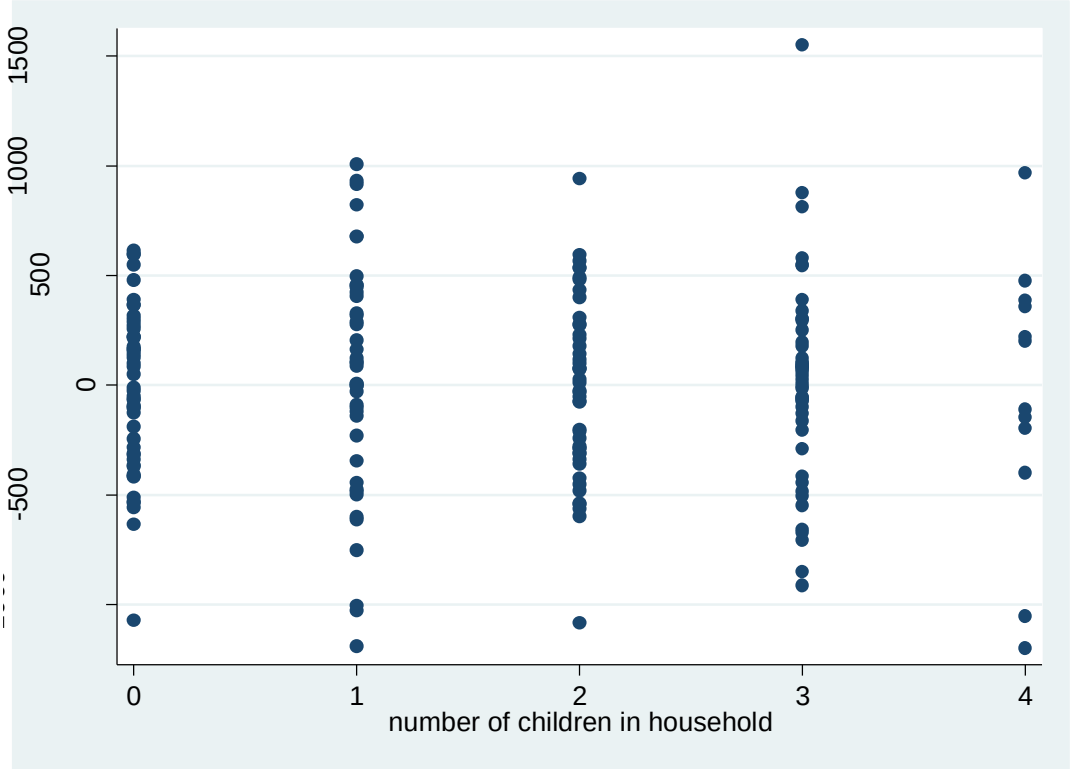
```
. reg miles income age kids

      Source |       SS       df       MS              Number of obs =     200
-------------+------------------------------           F(  3,   196) =   33.75
       Model |  20712796.2     3  6904265.41           Prob > F      =  0.0000
    Residual |  40098973.2   196  204586.598           R-squared     =  0.3406
-------------+------------------------------           Adj R-squared =  0.3305
       Total |  60811769.4   199  305586.781           Root MSE      =  452.31

------------------------------------------------------------------------------
       miles |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |   14.20133   1.800256     7.89   0.000     10.65097    17.75169
         age |   15.74092    3.75737     4.19   0.000      8.33086    23.15099
        kids |  -81.82642    27.1296    -3.02   0.003    -135.3298   -28.32302
       _cons |   -391.548   169.7752    -2.31   0.022    -726.3687   -56.72731
------------------------------------------------------------------------------
```

1. Briefly assess whether these results conform to your expectations.

2. To visually assess heteroskedasticity, we plot the residuals against each of the regressors, which yields


1500
1000
500
0
-500
20
40
60
80
100
120
annual income ($1000)


1500
1000
500
0
-500
20
30
40
50
60
average age of adult members of household



Based on these plots, what would you say about the likelihood of heteroskedasticity? Does this conform to what you would expect?

3. We next want to conduct a Breusch-Pagan test using the three regressors as potential variables to explain error variance. Initially, however, instead of regressing the squared residuals on the regressors we mistakenly use the actual (unsquared) residuals (*ehat*), yielding:

```
. reg ehat income age kids

      Source |       SS       df       MS              Number of obs =     200
-------------+------------------------------           F(  3,   196) =    0.00
       Model |           0     3           0           Prob > F      =  1.0000
    Residual |  40098973.4   196  204586.599           R-squared     =  0.0000
-------------+------------------------------           Adj R-squared = -0.0153
       Total |  40098973.4   199  201502.379           Root MSE      =  452.31

------------------------------------------------------------------------------
        ehat |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |   5.95e-09   1.800256     0.00   1.000     -3.55036     3.55036
         age |  -2.64e-08    3.75737    -0.00   1.000    -7.410064    7.410064
        kids |  -3.77e-07    27.1296    -0.00   1.000     -53.5034    53.50339
       _cons |  -1.23e-07   169.7752    -0.00   1.000    -334.8207    334.8207
------------------------------------------------------------------------------
```

Explain this result.

4. Correcting our procedure, we construct a variable *ehat2* for the squared residuals and regress it on the three regressors:

```
. reg ehat2 income age kids

      Source |       SS       df       MS              Number of obs =     200
-------------+------------------------------           F(  3,   196) =   11.09
       Model |  2.9205e+12     3  9.7349e+11           Prob > F      =  0.0000
    Residual |  1.7210e+13   196  8.7806e+10           R-squared     =  0.1451
-------------+------------------------------           Adj R-squared =  0.1320
       Total |  2.0130e+13   199  1.0116e+11           Root MSE      =  3.0e+05

------------------------------------------------------------------------------
       ehat2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |   6194.121   1179.389     5.25   0.000     3868.199    8520.043
         age |  -7512.891   2461.539    -3.05   0.003    -12367.39   -2658.388
        kids |   37255.68   17773.22     2.10   0.037     2204.383    72306.98
       _cons |   64235.25   111223.6     0.58   0.564    -155113.4    283583.9
------------------------------------------------------------------------------
```

Perform the Breusch-Pagan χ^2 test and interpret your conclusion about heteroskedasticity.

5. Based on your results, what are the properties of your OLS estimates? (Are they consistent? Are they efficient?) What (if any) further statistical analysis do you recommend?