Economics 312 — Spring 2020
Daily Problem #10 — February 13

This daily problem is a little harder than usual because it involves manipulating matrices. It's OK to skip this one if you find it too mathematically challenging. Appendix E of Wooldridge will be useful.

Our model is, as usual, $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$.

We usually assume that the error terms u_i are uncorrelated and homoskedastic.

1. Show that in matrix form this corresponds to $\text{var}(\mathbf{u}) = E(\mathbf{uu}') = \sigma^2\mathbf{I}_n$, where $\mathbf{I}_n$ is the identity matrix of order n: $\mathbf{I}_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$. (A matrix that is a constant times the identity matrix is called a "scalar matrix" because it essentially acts like a scalar: multiplying it by any matrix is equivalent to multiplying by a scalar.)

2. Given this assumption of a scalar covariance matrix for the error, show that the covariance matrix of the OLS coefficient estimator $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$ is $\sigma^2(\mathbf{X}'\mathbf{X})^{-1}$. Hint: Plug in $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$ and solve to get $\hat{\boldsymbol{\beta}} = \boldsymbol{\beta} + (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{u}$. Then recall that $\text{var}(\hat{\boldsymbol{\beta}}) = E\left[(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})'\right]$. (All variances and expectations conditional on $\mathbf{X}$.)