Economics 304 **Fall 2017**
Problem Set #1 **Due: September 15**

1. This is an algebraic problem pertaining to the economy of Reedistan, which behaves according to the Solow growth model. Suppose that the production function in Reedistan is

$$Y = F(K, AL) = K^{1/3}(AL)^{2/3}.$$

a. Show that the intensive form of this production function is $y = k^{1/3}$.

b. Solve for the steady-state values $\bar{k}$ and $\bar{y}$ as functions of s, δ, n, and a.

c. Suppose that $\delta = 0.06$, $n = 0.02$, and $a = 0.01$. Calculate the values of $\bar{k}$, $\bar{y}$, and $\bar{c}$ for $s =$ 0.04, 0.09, 0.16, 0.25, 0.36, and 0.49. Which of these values of s is the "best"?

d. The laws of calculus tell us that the marginal product of capital for this production function depends on k according to $MPK = \frac{1}{3}k^{-2/3}$. Using this formula, calculate the Golden Rule values of s, $\bar{k}$, and $\bar{c}$ for the parameters given in part c. Is this consistent with the evidence from part c?

e. Suppose that (with the other parameters as in part c) the saving rate is initially 0.09 and Reedistan is in a steady-state equilibrium. Shortly after a revolution brings him to power in Year 1, Grand Ayatollah Mohammed Al-Kroger decrees convincingly that excessive consumption is sinful and as a result the saving rate increases to 0.36. Assume that the capital stock in any year is determined by the *previous* year's saving, so that the change in saving in year one does not affect the capital stock until year 2:

$$k_2 - k_1 = \Delta k_2 = s_1 f(k_1) - (\delta + n + a)k_1.$$

How much will k and y increase in year 2 as a result of the increase in the saving rate that occurred in year 1? What percentage of the gap between the old and new steady state values of y has been made up in year 2? (This percentage is called the "rate of convergence.") About how many years will it take for y to travel halfway to the new steady state? [You might find a spreadsheet helpful in doing the calculations for this problem.]

2. Endogenous growth models incorporate an expanded concept of capital that has non-diminishing marginal returns, often justified as knowledge capital or human capital. Consider a simple production function $Y = BK$, in which K is the stock of this expanded capital, B is a constant, and Y is output. We assume that the labor force is constant, so we don't need to include it in and production function and we can do the familiar Solow diagram in terms of capital letters rather than the intensive form. There is no "exogenous" growth due to population or exogenous technology in

this model. Suppose that the accumulation of capital is as in the Solow model: $\Delta K = sY - \delta K$. You should assume that $sB > \delta$.

a. Find an expression for ΔK in terms of K and the parameters of the model (B, s, and δ).

b. Show a graph analogous to the one we used for the Solow model (except with K on the horizontal axis instead of k) and use it to describe the dynamic behavior of K.

c. What is the growth rate of K in this model? What is the growth rate of Y?

d. Is there a process of "convergence" to a steady-state path in which lower-income countries will grow faster until they catch up, or is the economy always growing at the same rate regardless of the initial level of K?

e. How, if at all, is the growth rate affected by a change in the saving rate?

f. Contrast these conclusions with those of the version of the Solow model (with diminishing returns to scale) in which the labor force and exogenous technology do not grow.