Econ 201: Introduction to Economic Analysis

September 25 Lecture: Production

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Daily dose of The Far Side

www.thefarside.com

Darrell suspected someone had once again slipped him a trick spoon with the concave side reversed.

Preview of this class session

- We analyze a simple production setup with one output and two inputs
- One input is fixed in short run (“capital”) the other variable (“labor”)
- Production function relates output to inputs
- We define total, average, and marginal products of labor
- Isoquants are production equivalent to indifference curves
- Long-run cost minimization leads to tangency of isoquant with isocost line

Digression on the Nature of Production, Output, and Inputs

Stocks and flows

- **Stocks**: Variables measured "at an instant" with no time in units
  - Amount of water in bathtub (gallons)
  - Number of students at Reed
- **Flows**: Variables measured "over time"
  - Changes in stocks are flows: inflow to tub through faucet (gallons per minute) and outflow through drain (gallons per minute)
- Production process is a flow that accumulates over time
  - 15 widgets per day, perhaps

Intermediate goods vs. factors of production

- Simple production process: two kinds of inputs
- **Intermediate good inputs** are fully consumed during the production process
  - Making coleslaw uses up the cabbage completely
  - Mowing the lawn uses up gasoline to run mower
- **Factors of production**: only services are used for period of time
  - **Labor**: My wife is not fully used up by making coleslaw, the lawn crew still exists after mowing is finished
  - **Capital**: Food processor and lawn mower are not fully consumed
- We will ignore intermediate goods and focus on factors of production as inputs

Aggregating labor and capital

- There are many different kinds of “labor” and “capital”
- In complex model, many different labor and capital inputs to production
- In simple model, we aggregate into single labor input and single capital input
  - Combine all kinds of labor into single weighted “index” of labor input (weights based on wage?)
  - Combine all kinds of capital (machines) into single weighted index of capital input (weights based on price?)

Nature of labor and capital as inputs

- Labor must be rented, not purchased
  - Labor input is measured in "person-hours" of (quality adjusted) **labor services**
- Capital input = services of machines, not machines themselves
- Capital can be purchased or rented
  - Most firms own lots of machines, factories
  - Two distinct roles in production: capital and entrepreneurship
  - We separate these conceptually by treating capital input is a flow of **capital services**, measured in (quality-adjusted) "machine-hours"
  - Think of firm as owning machines and renting to itself for production

Production

- The **production process** combines labor services and capital services (and intermediate goods that we neglect) to produce output
- Making coleslaw:
  - An hour of my wife's labor input
  - Ten minutes of food-processor services, plus some knife services, container services, etc.
  - Output = bowl of coleslaw
- Mowing lawn:
  - 20 minutes of three-man lawn crew = one person-hour of labor
  - 20 minutes of lawn-mower services, trimmer services, leaf-blower services
  - Output = nice-looking lawn with grass of appropriate length

Output, Inputs, and the Production Function

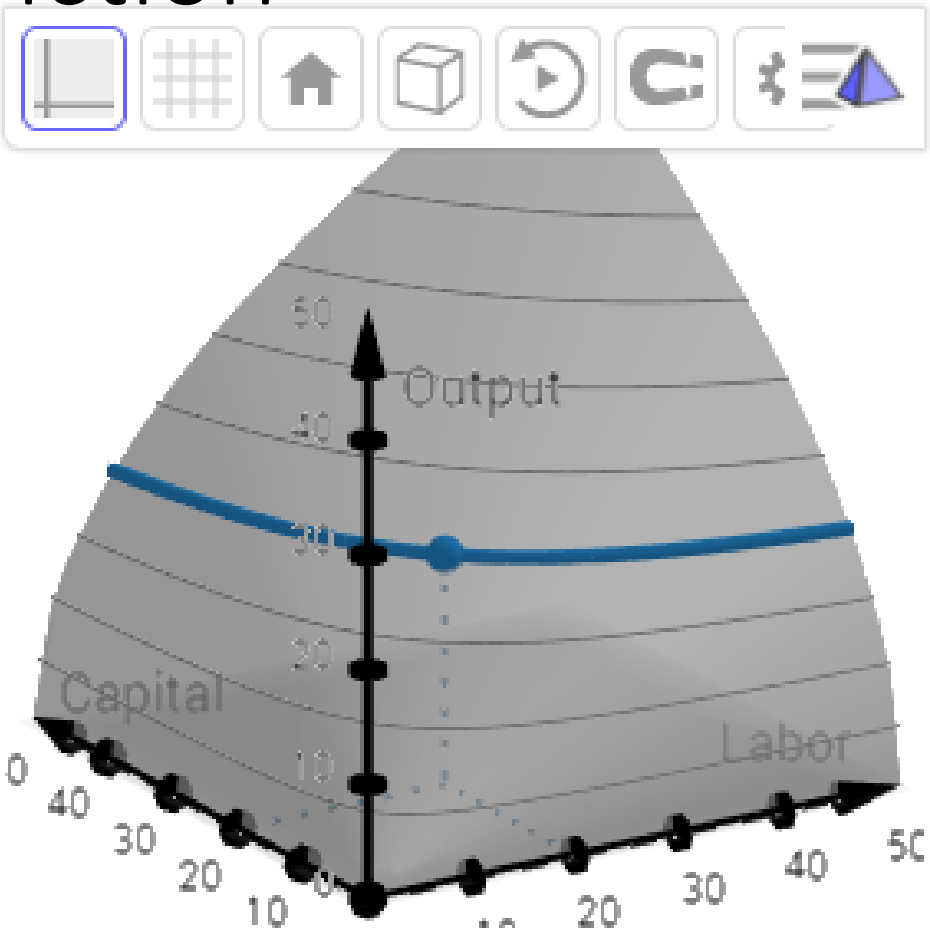
Fundamental concepts of economic production by firms

Simplifying assumptions

- One output good Q
- One fixed factor K
  - We call it capital, but some labor inputs are also fixed in short run
  - In case study, all inputs except feed are fixed
- One variable input factor L
  - Again, we call labor, but some capital inputs are variable
  - In case study, feed is variable factor
- Production function: $Q = f(K, L)$
  - Q increases with increase in either K or L

Geometry of production function

- Production function: $Q = f(K, L)$
- Output Q is measured vertically
- Labor input L back to right; capital input K back to left
- Looks like utility mountain
  - In this case, we *can* attach numbers to vertical axis: units of output produced Q
- We'll use the contour lines later



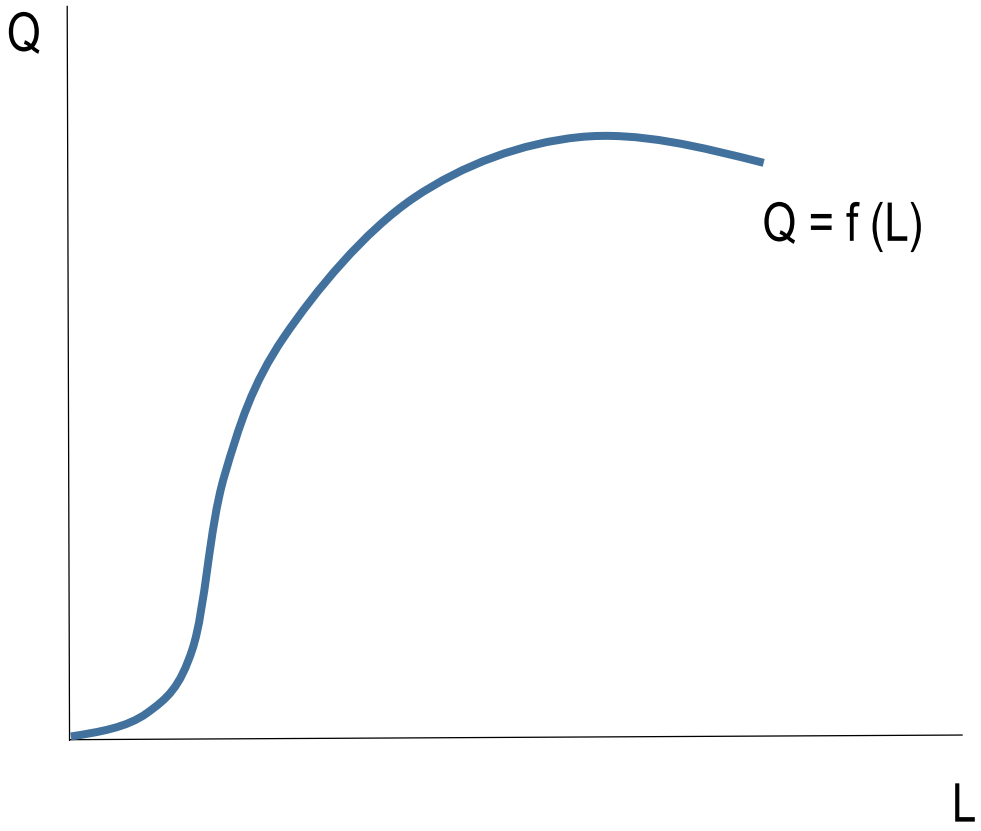
https://www.econgraphs.org/graphs/micro/producer_theory/production_and_cost/cobb_douglas_production_3d

Fixed and variable factors

- Short run: Input of **fixed factors** cannot change
  - Firm cannot build more factories; college cannot fire tenured faculty; dairy farmers cannot fire family members working on farm
- We usually call the fixed factor "capital"
- Short-run production function: $Q = f\left(\bar{K}, L\right)$
- **Variable factors** can change in short run (or long run): labor on short-term contracts
  - We call variable factor "labor" (though it's feed in the case)
  - Some kinds of capital can be varied in short run
- Long run: All factors can be varied
  - Even fixed capital can change in long run

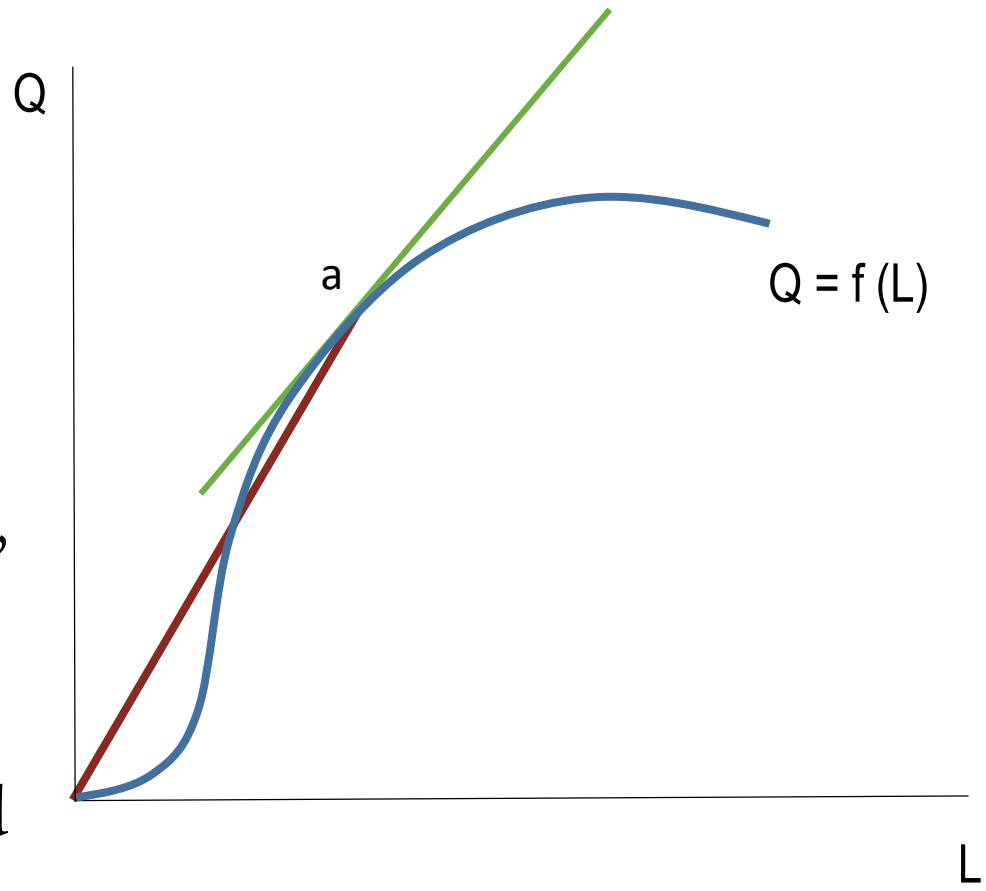
Short-run production function

- Slice 3-dimensional long-run production function from left-front to back-right with vertical plane
- Pattern traced out on plane is **short-run production function** for given capital $\bar{K}$
- $Q = f\left(\bar{K}, L\right) = f(L)$
- Upward-sloping at decreasing rate
  - "**Marginal product**" is positive, but eventually declining
  - Law of diminishing marginal productivity



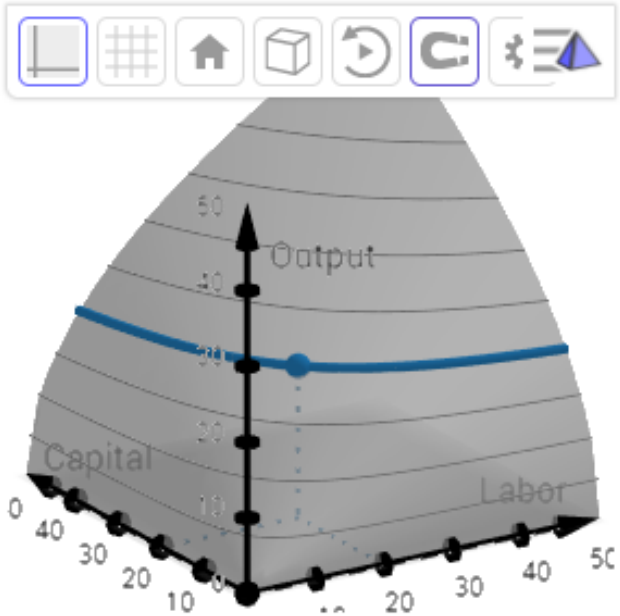
Total, average, and marginal product in short run

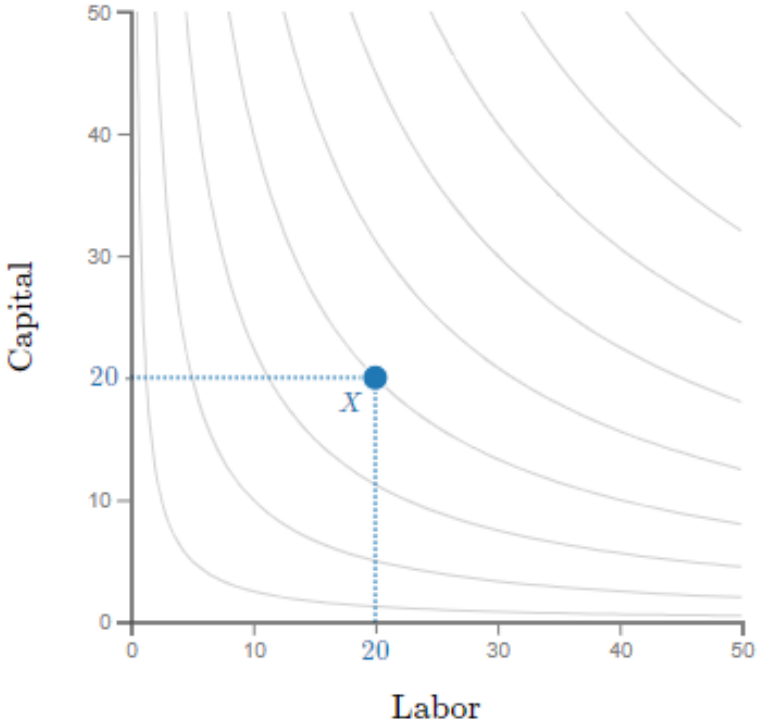
- **Total product**: $Q = f\left(\bar{K}, L\right) = f\left(L\right)$
- **Average product**: Q/L
  - Amount produced by "average worker"
  - Geometrically: slope of **line connecting point to origin**
- **Marginal product:** $\Delta Q \,/\, \Delta L$
  - Amount produced by "marginal worker"
  - Geometrically: slope of **tangent line** at point
- Increase in capital would shift upward



Long-run production and choosing the cost-minimizing combination of inputs

Isoquants and long-run production



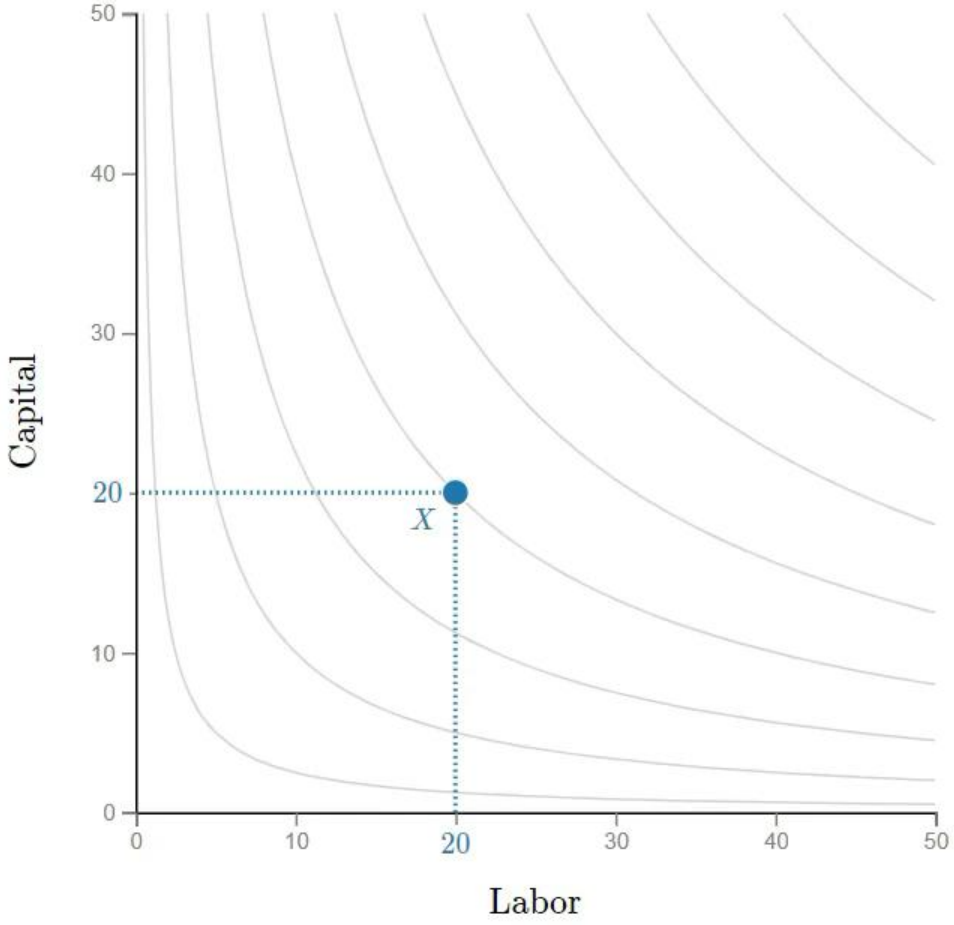


- Isoquants are contour lines of production function: $\bar{Q} = f(K, L)$
- Different combinations of K and L that can produce $\bar{Q}$

https://www.econgraphs.org/graphs/micro/producer_theory/production_and_cost/cobb_douglas_production_3d

Substitution among inputs

- Firm can produce given quantity of output in many ways
  - With a lot of labor and a little capital
  - With a lot of capital and a little labor
  - With medium amounts of both inputs
- Absolute slope of isoquant is margin rate of technical substitution
  - MRTS = $-\frac{\Delta K}{\Delta L} = -\frac{\Delta Q / \Delta L}{\Delta Q / \Delta K} = \frac{MP_L}{MP_K}$
- MRTS decreases with more L, less K



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Costs in the long run

- Cost of one person-hour of labor services = W
- Cost of one machine-hour of capital services?
  - What would one machine-hour rent for?
  - Must compensate owner for depreciation δ and forgone interest r
  - Per year: rental price of one dollar of capital = $r + \delta$
  - Annual rental price of one machine = $P_K \times (r + \delta) = R$
  - Can pro-rate to one hour (or measure labor in person-years)
- Total cost = $C = (W \times L) + (R \times K)$

Iso-cost lines

- **Iso-cost lines** connect points with same total cost:

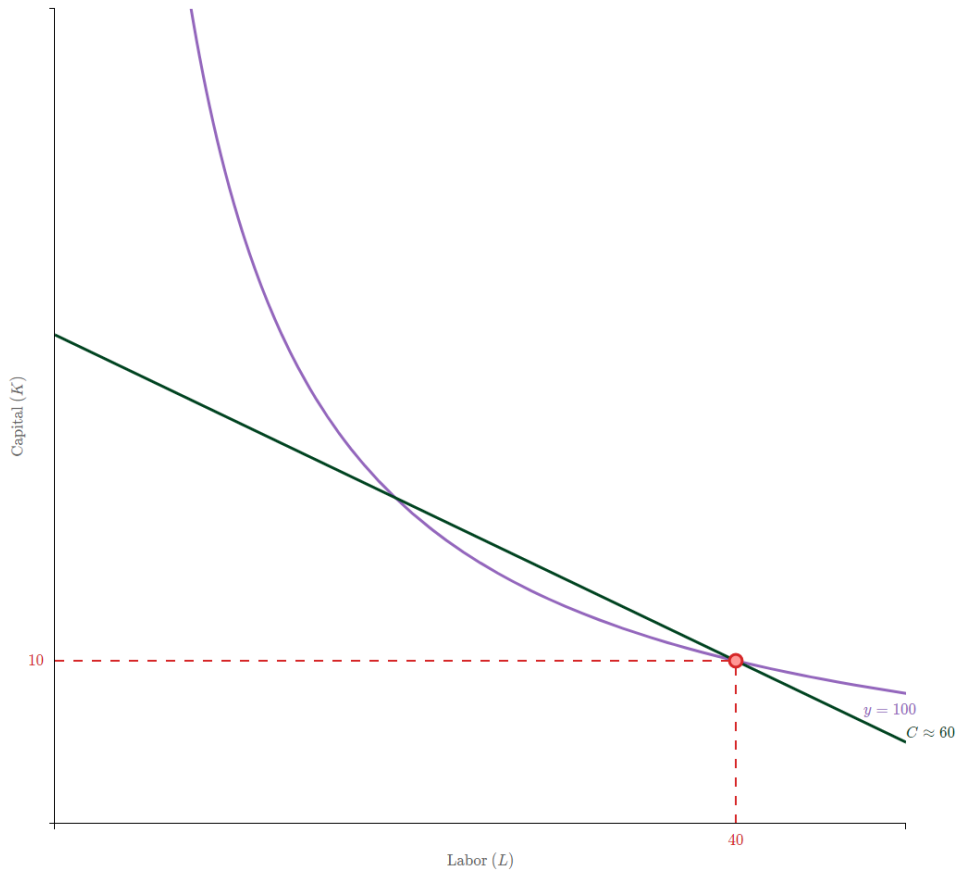
$$\bar{C} = (W \times L) + (R \times K)$$

- Solve for K (on vertical axis):

$$K = \frac{\bar{C}}{R} - \frac{W}{R} L$$

- Line with intercept $\frac{\bar{C}}{R}$ and slope $-\frac{W}{R}$

- $W = 1,\ R = 2,\ \bar{C} = 1 \times 40 + 2 \times 10 = 60$



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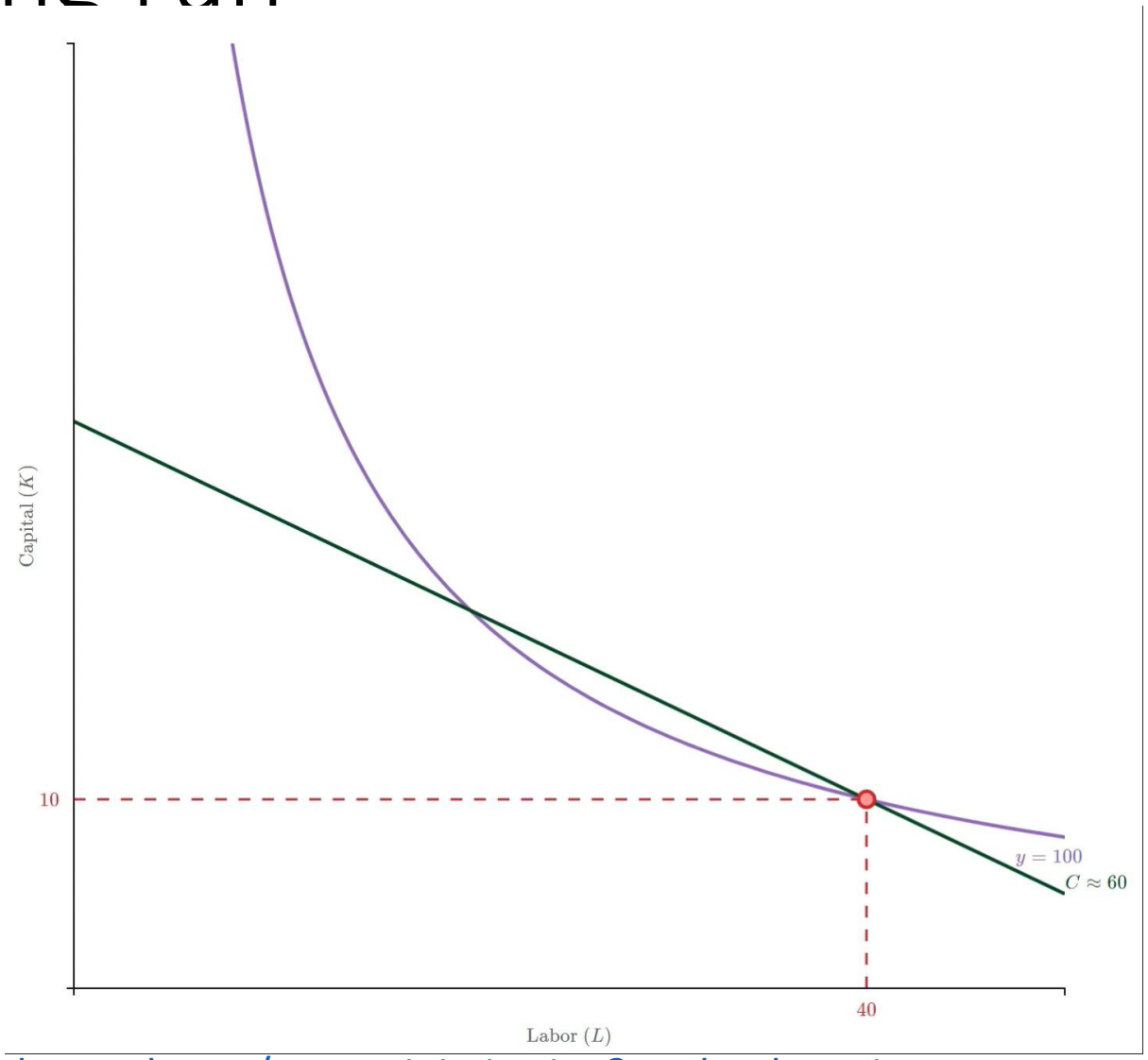
Cost minimization in long run

- At the initial point, firm incurs cost of 60 to produce output of 100
- There are points inside the little crescent that cost less than 60 and produce at least 100
- Using more capital and less labor lowers cost of producing 100 units



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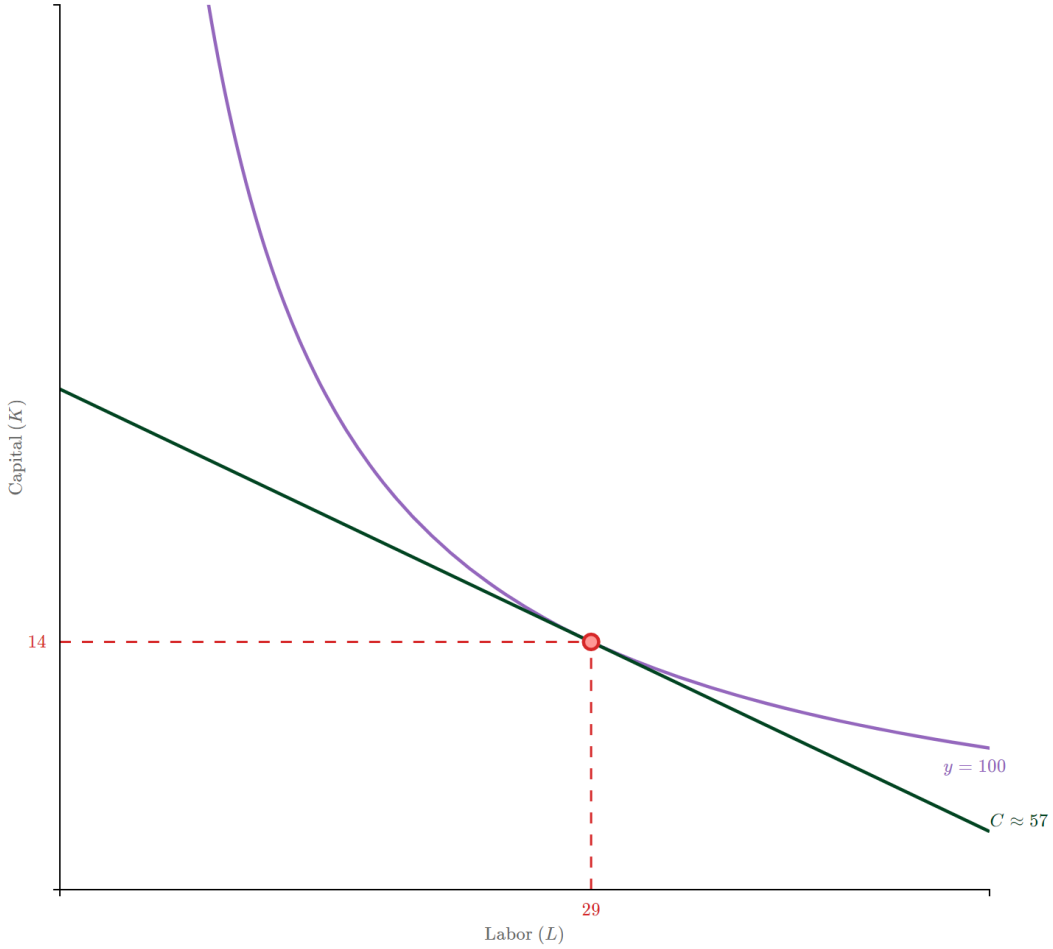
At tangency: costs are minimized for given Q

- MRTS = absolute slope of iso-cost line
  - $\mathrm{MP_L}/\mathrm{MP_K} = W/R$ or
  - $\mathrm{MP_L}/W = \mathrm{MP_K}/R$ or
  - $W/\mathrm{MP_L} = R/\mathrm{MP_K}$

- Marginal cost of output using labor = marginal cost of output using capital
- Otherwise could lower costs by substituting input with lower cost for one with higher cost



Returns to scale in long run

- **Constant returns to scale**
  - 10% increase in ALL inputs raises output by EXACTLY 10%
- **Increasing returns to scale**
  - 10% increase in all inputs raises output by MORE than 10%
- **Decreasing returns to scale**
  - 10% increase in all inputs raises output by LESS than 10%
  - Not the same as diminishing marginal returns/product
    - Returns to scale: What happens when ALL inputs increase?
    - Marginal returns: What happens when ONE input increases with others held constant?

Technological change

- Improvement in technology of production:
  - More output from same amount of inputs, or
  - Same output using less inputs
- Shifts production function upward
- Shifts isoquants down to the left
  - Any give quantity of output can be produced with less labor, less capital, or (usually) both
- Could change shape if technological change is "biased" toward labor or capital
- Whole class devoted to economics of technological change later

Review

- We ignore intermediate inputs and focus on factors: labor and capital
- Capital is fixed in short run; labor is variable
- Production function relates output to inputs
- Total, average, and marginal product
- Isoquants are production equivalent to indifference curves
- Long-run cost minimization leads to tangency of isoquant with iso-cost line

Daily diversion: Another bad economist joke

A guy walks into a Washington D.C. curio shop. After browsing, he comes across an exquisite brass rat.

“What a great gag gift,” he thinks to himself. After dickering with the shopkeeper over the price, the man purchases the rat and leaves.

As he's walking down the street, he hears scurrying sounds behind him. Stopping and looking around, he sees hundreds, then thousands of rats pouring out of alleys and stairwells into the street behind him. In a panic, he runs down the street with the rats not far behind.

The street ends at a pier. He runs to the end of the pier and heaves the brass rat into the Potomac River. All of the rats scurry past him into the river, where they drown.

After breathing a sigh of relief and wiping his brow, the man heads back to the curio shop, finds the shopkeeper, and asks, “Do you have any brass economists?”

--Taken from Jeff Thredgold, *On the One Hand: The Economist's Joke Book.*

What comes next?

- Next class: Cost functions!
  - Relating total, average, and marginal cost to amount of output produced in short run or long run
- Cases of the day: Continuing with dairy farms case
  - Do not turn in the cases day by day, but do them and save them up
  - This becomes Problem Set #4, which is due on Wednesday, September 30
- Exam ahead!
  - The first midterm exam will be held on Monday, October 5
  - More details as the time gets closer