PopBio Help

This document briefly describes the equations used by each of the simulations in the PopBio program. A more detailed application manual is planned for a future release. In the mean time, if you have a question about the PopBio program that is not answered here please contact Ethan Benatan at benatan@reed.edu.

Population growth

Variables

- x = [population]
- x_0 = [initial population]
- r = [rate of increase]
- K = [carrying capacity]
- L = [growth time lag]

Equations

Exponential growth

The basic assumption of population growth modeling is that the rate of population growth is proportional to the current population – that is,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx \tag{1}$$

The solution to (1) is

$$x(t) = x_0 e^{rt} \tag{2}$$

Equation (2) is used for computing $x(t)$ deterministically; the stochastic mode uses equation (1), recalculating the value of r at each time step.

Logistic growth

To incorporate the carrying capacity, we add a term to $\mathrm{d}x/\mathrm{d}t$:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1 - \frac{x}{K}\right) \tag{3}$$

The solution to this equation is

$$x(t) = \frac{Kx_0 e^{rt}}{K + x_0\left(e^{rt} - 1\right)} \tag{4}$$

As with exponential growth, the deterministic mode uses the second equation, while the stochastic mode uses the first, recalculating K and r at every time step. If the growth lag in a stochastic calculation is non-zero, then for $t \geq L$, $x(t - L)$ is used in place of x in equation (3).

Predator-Prey

Variables

- $N =$ [prey population]
- $P =$ [predator population]
- $a =$ [prey birth rate]
- $b =$ [prey capture probability]
- $c =$ [predator energy efficiency] $\cdot\, b$
- $d =$ [predator death rate]
- $K =$ [prey carrying capacity]
- $e =$ [prey critical density]
- $m =$ [maximum predator attack rate]
- $W =$ [1/2 attack rate population]
- $w =$ [1/2 attack rate density]

Lotka-Volterra model

$$\begin{aligned}
\mathrm{d}N/\mathrm{d}t &= aN - bNP \\
\mathrm{d}P/\mathrm{d}t &= cNP - dP
\end{aligned}$$

Leslie model

$$\begin{aligned}
\mathrm{d}N/\mathrm{d}t &= aN(1 - N/K) - bNP \\
\mathrm{d}P/\mathrm{d}t &= cP(1 - eP/N)
\end{aligned}$$

Holling models

$$\begin{aligned}
&\mathrm{d}N/\mathrm{d}t = aN(1 - N/K) - b(N,P)P \\
&\mathrm{d}P/\mathrm{d}t = c\,b(N,P)P - dP \\
&b(N,P) = mN/(W + N) && \text{(Holling I functional response)} \\
&b(N,P) = mN/(wP + N) && \text{(Holling II functional response)}
\end{aligned}$$

Population Genetics

Variables

- $p =$ [A allele frequency]
- $S =$ [Population size]
- $w_{11} =$ [Genotype AA fitness]
- $w_{12} =$ [Genotype Aa fitness]
- $w_{22} =$ [Genotype aa fitness]

Equations

Given p_b (with $q_n \equiv 1 - p_n$), compute p_{n+1} using the following equation:

$$p_{n+1} = p_n q_n \frac{(w_{11} - w_{12})p_n + (w_{12} - w_{22})q_n}{w_{11}\, p_n^2 + 2w_{12}\, p_n q_n + w_{22}\, q_n^2} + r_n \sqrt{\frac{3p_n q_n}{2N}}$$

If genetic drift is enabled, then for each n, r_n is a random number in the range $(-1, 1)$; otherwise, $r_n = 0$ for all n.

Life Table

Variables

- $N =$ [number of age groups]
- $x =$ [age group index between zero and $N-1$ inclusive]
- $n_x =$ [initial population of age group x]
- $P_x =$ [probability of an individual surviving to age group x]
- $m_x =$ [fecundity of age group x]

Equations

The columns of the Life Table output matrix are computed using the following equations.

$$p_x = P_{x+1}/P_x$$
$$q_x = 1 - \frac{p_{x+1}}{p_x}$$
$$d_x = P_x - P_{x+1}$$
$$e_x = P_x^{-1} \sum_{j=x+1}^{j<N} P_j$$

The Age Structure animation is computed in the following way. At every time interval the population in age group x advances into age group $x+1$ and size of the advancing population is given by

$$n_{x+1_{t+1}} = n_{x_t} p_{x_t}$$

After each age group has been advanced, the first age group is populated according to the equation

$$n_{0_{t+1}} = \sum_{j=0}^{j<N} n_{x_t} m_x$$

The survivorship curve visualizes the probability that individuals will survive into each age group and the total population curve shows $\sum n_x$ over time.

Competition

Variables

- $x =$ [species population]
- $r =$ [rate of increase]
- $K =$ [carrying capacity]
- $\alpha_{12} =$ [species 1 competition coefficient]
- $\alpha_{21} =$ [species 2 competition coefficient]

Equations

$$\frac{\mathrm{d}x_1}{\mathrm{d}t} = r_1 x_1 \left(1 - \frac{x_1 + \alpha_{12} x_2}{K_1}\right)$$

$$\frac{\mathrm{d}x_2}{\mathrm{d}t} = r_2 x_2 \left(1 - \frac{x_2 + \alpha_{21} x_1}{K_2}\right)$$