DIMENSIONAL ANALYSIS, THE METRIC SYSTEM AND SIGNIFICANT FIGURES

Exponents

The rules:

What is an exponent?

10^6 means 10 times itself 6 times.

Multiplying numbers with exponents:

$$a^r \cdot a^s = a^{r+s}$$

this means:

$$10^3 \cdot 10^4 = 10^{3+4} = 10^7$$

$$10^{-3} \cdot 10^4 = 10^{-3+4} = 10^1 = 10$$

but what is the meaning of 10^{-3}?

$$10^{-3} = \frac{1}{10^3}$$

Dividing numbers with exponents:

$$\frac{a^r}{a^s} = a^{r-s}$$

$$10^{-3} \cdot 10^4 = \frac{10^4}{10^3} = \frac{10^{3+1}}{10^3} = 10^1 = 10 \quad \text{(write it out with 10s)}$$

Raising numbers with exponents to other powers:

$$(a^r)^s = a^{rs}$$

$$(10^4)^3 = 10^{(4 \cdot 3)} = 10^{12}$$

why? $10^4 \cdot 10^4 \cdot 10^4 = 10^{4+4+4}$

and

$$(10^4)^{-3} = 10^{(4 \cdot (-3))} = 10^{-12} = \frac{1}{10^{12}}$$

Ok, now let's look at the problem from the diagnostic:

$$\frac{10^8 \times 10^{-1}}{10^{10}} = \quad ? \qquad\qquad =10^{-3} \quad \text{(solve it a few ways)}$$

Do some other examples.... $(x^{-1})^{-1} = x$ $\left(\frac{a}{b}\right)^{-r} = \left(\frac{b}{a}\right)^{r}$

And, $\sqrt{a} = a^{\frac{1}{2}}$ (do some examples)

Less common, but also true is $\sqrt[3]{a} = a^{\frac{1}{3}}$, etc.

There's one more rule you should know about exponents:

$a^0 = 1$

why? Well, $\frac{10^r}{10^r} = 1$, right? and $\frac{10^r}{10^r} = 10^{r-r} = 10^0$ so this must also be = 1

Scientific Notation

Really just a way of dealing with numbers like 40,000,000,000,000,000 or 1/30,000,000,000

4 x 10^{15} and 3 x 10^{-11} are so much easier, especially once you start multiplying and dividing!!!

Do an example

Or even adding and subtracting… do an example

In scientific notation, any number not between 1 and 10 should be converted:

62,500,000 = 6.25 x 10^7

0.0023 = 2.3 x 10^{-3}

So now, (4.19 x 10^7) (7.08 x 10^2) = 29.7 x 10^9 = 2.97 x 10^{10}

This brings up the issue of significant digits. My calculator says 4.19 x 7.08 = 29.6652

How do you know how many numbers to drag along with you?

Significant Figures

Do a bunch of examples of numbers and how many sig figs they have

26 → 2	2006 → 4	2600 → 2
0.4 → 1	0.00004 → 1	0.400 → 3
7400 → 2	7400. → 4	

For multiplication and division, do all your steps, then look at what you started with. Whichever number has the fewest, that's how many your answer has.

Also, don't round off (up) till the end!

For subtraction and addition, the rules are a little more complex… but here's a couple of ways to look at it:

You can't add a new decimal place. If one is lost, a sigfig is lost…

213.2 – 172.5 = 40.7

You can't add precision.

130.1 + 0.002 = 130.1

Nor do you need to take it away needlessly.

162-3 = 159 (accuracy here is in the "ones" column)

So, how do you know how many digits to write down when you MAKE a measurement?

Digital?

- Steady? Write down all numbers
- Fluctuating? Take all steady numbers and estimate the next one smaller

Analog?

- Take all numbers that have a scale (tick mark, line, etc.) and estimate the next one smaller

There are a few special cases… IGNORE the sig figs of

1. constants (pi, speed of light, etc.)
2. ratios of integers (1:2 molar ratio)

3. defined numbers (1 hogshead = 63 gallons)

Now, let's look at the <u>Metric System</u>:

Just go straight off the handout…

Ok, now we're ready to combine them.

Let's return to the question on the diagnostic:

If 1 cm = 0.01 m, then 1 cm^3 = how many m^3?

So, if 1 cm = $10^{-2} m$, then $(1\ cm)^3$ = $\left(10^{-2} m\right)^3$ = $10^{-6} m^3$

Now, if a cell is 10 μm wide, 10 μm tall and 20 μm long, how many cubic meters of volume?

10 μm x 10 μm x 20 μm = 2,000 μm^3 = 2 x 10^3 μm^3

1 μm = 10^{-6} m from the handout, so $(1\ \mu m)^3$ = $(10^{-6}\ m)^3$ = ? = 10^{-18} m^3

So, 2 x 10^3 μm^3 = (2 x 10^3)(10^{-18} m^3) = ? = 2 x 10^{-15} m^3

<u>Dimensional Analysis</u>

Remember the question from the diagnostic:
If 1 hogshead = 0.5 butt, and 1 hogshead = 63 gallons, how many gallons are in 1 butt?

How do we get from one unit of measurement to another?

1m = 100 cm

So if we have 44 cm, how many meters is it?

THE TRICK: make a conversion multiplier: $\frac{1\,\mathrm{m}}{100\,\mathrm{cm}}$

So 44 cm $\left(\frac{1\,\mathrm{m}}{100\,\mathrm{cm}}\right)$ = $\frac{44}{100}$ m = 0.44 m

The reason we can do this is that the conversion multiplier = 1

Other examples of conversion multipliers:

$$\left(\frac{1\,\text{L}}{10^{-3}\,\text{m}^3}\right) \qquad \left(\frac{1\,\text{atm}}{1.013\times10^5\,\text{Pa}}\right) \qquad \left(\frac{1\,\text{Joule}}{1\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}}\right)$$

you can do this for any a=b to make a conversion factor of $\left(\frac{\text{a}}{\text{b}}\right)$ or $\left(\frac{\text{b}}{\text{a}}\right)$ to change the units

Units can save your butt by alerting you to a mistake!

Never, ever write numbers that have units without the units!!!!

Units can save your butt by what they tell you (what's a joule? An acceleration is m/s2, force is mass x acceleration (kg.m/s2) and work is force over a distance (kg.m2/s2))

Always think about the units, don't just write them down!!!

And now for the diagnostic question:

If 1 hogshead = 0.5 butt, and 1 hogshead = 63 gallons, how many gallons are in 1 butt?

The problem gives us two conversion factors: $\left(\frac{1\,\text{hogshead}}{0.5\,\text{butt}}\right)$ and $\left(\frac{1\,\text{hogshead}}{63\,\text{gallons}}\right)$

So, we want 1 butt = ? gallons:

→ (1 butt) $\left(\frac{1\,\text{hogshead}}{0.5\,\text{butt}}\right)\left(\frac{63\,\text{gallons}}{1\,\text{hogshead}}\right) = \frac{63}{0.5}$ gallons = 126 gallons

Tracking the units in a calculation is called Dimensional Analysis:

1) What's the frequency of light that has a wavelength of 671 nm?

First, 671 nm = 671 x 10^{-9} m = 6.71 x 10^{-7} m

Next, $\nu = \frac{c}{\lambda} = \frac{3\times10^8\,\frac{\text{m}}{\text{s}}}{6.71\times10^{-7}\,\text{m}}$ = 4.47 x 10^{14} s^{-1}

2) If an object has a mass of 15.0 g and a volume of 10.0 cm^3, what's its density?

Density = $\frac{\text{mass}}{\text{volume}} = \frac{15.0\,\text{g}}{10.0\,\text{cm}^3} = 1.50\,\frac{\text{g}}{\text{cm}^3}$

3) Calculate the quantity of heat that must be transferred to 15.0 g of water to raise its temperature from 20.0 °C to 50.0 °C?

(Water has a specific heat of 4.18 $\frac{\text{J}}{\text{g} \cdot {}^\circ\text{C}}$)

Heat transferred = (specific heat)(mass)(ΔT)

ΔT = 30.0 °C

So, Heat transferred = (4.18 $\frac{\text{J}}{\text{g} \cdot {}^\circ\text{C}}$)(15.0 g)(30.0 °C) = ? = 1.88×10^3 J