4.3. Explaining the Dynamics: Insights using the Stuart-Landau Model

Figure 4.4: Figure depicting the bifurcation the optoelectronic oscillator with a perturbing feedback loop exhibits as $\kappa$ is increased.

red (thicker part of the ellipse). Part (a) shows the solutions when the feedback loop has maximal attenuation. Here, only one solution is possible, and thus this is the waveform observed in the lab. As $\kappa$ increases, the percentage of the ellipse that is stable decreases, and more solutions become possible. For example, at $\kappa = .5$ which is shown in part (b), five solutions are possible, two of which are stable. One of these two stable solutions is the original solution. Since the system will not naturally deviate from a stable solution, the dynamics do not change in this regime. As $\kappa$ increases further, eventually the original solution becomes unstable. The result of this instability is that the observed solution diverges from the original solution, and eventually reaches a part of phase space in close proximity to a stable solution, to which it is then attracted (part c). Further increasing $\kappa$ results in higher amplitude solutions (part d), but there are no more bifurcations since the second solution remains stable throughout the remaining change in attenuation.\(^1\)

The amplitude and frequency values corresponding to Fig. 4.4, as predicted by the Stuart-Landau equation, are shown in Fig. 4.5. The relation between the non-dimensionalized factors present in the Stuart-Landau equation and their tunable

\[^1\)It is speculated that the observed bifurcation is an unstable torus bifurcation of a limit cycle, also known as a subcritical Hopf bifurcation. This bifurcation is classified by an unstable torus that surrounds the stable limit cycle. At the bifurcation point, this torus and the limit cycle merge. The limit cycle becomes unstable, and solutions that are perturbed diverge from their original points [19].