1. For each example, either verify that it is a member of the exponential family and identify the sufficient statistics, or erxplain why it is not a member of the exponential family.

   (a) $X_1, X_2, \dots X_n$ are an IID sample from a Poisson(μ) distribution.

   (b) $X_1, X_2, \dots X_n$ are an IID sample from a Gamma(α, β) distribution.

   (c) $X_1, X_2, \dots X_n$ are an IID sample from a Uniform($0, \theta$) distribution.

   (d) $X_1, X_2, \dots X_n$ are an IID sample from the 5 parameter mixture distribution with density
   $$f(x) = p\, n(x \mid \mu_1, \sigma_1^2) + (1-p)\, n(x \mid \mu_2, \sigma_2^2)$$
   where $n(x \mid \mu, \sigma^2)$ is the Normal(μ, σ^2) density.

   (e) X is a Negative Binomial(r, p) random variable.

   (f) $X_1, X_2, \dots X_n$ are an IID sample from a Cauchy(θ) distribution.

2. Let X be a Negative Binomial(r, p) random variable. Recall that if $X = k$ then there were $r + k$ independent Bernoulli(p) trials with r 1's and k 0's.

   (a) Show that the outcome of the first trial, call it Y_1, is an unbiased estimator for p.

   (b) Use the Rao-Blackwell theorem to find a better unbiased estimator.

3. Let $X_1, X_2, \dots X_n$ be an IID sample from a Poisson(μ) distribution.
   Note that $\mathbb{P}(X_i = 0) = e^{-\mu}$.

   (a) What is the MLE for $\theta = e^{-\mu}$? What is its variance (exactly if possible, otherwise approximately)?

   (b) Define $Y_i = 1$ if $X_i = 0$, and 0 else. Show that $\bar{Y}$ is unbiased for θ. What is its variance?

   (c) Evaluate $\hat{\theta} = \mathbb{E}(Y_1 | \sum X_i)$ and find its variance.