1. Let $X_1, X_2, \ldots X_n$ be IID Poisson(θ). Find 95% confidence/probability intervals by the following methods for Ladislaus von Bortkiewicz's data on deaths by horse kick for Prussian Cavalry units (*Das Gesetz der kleinen Zahlen*, 1898). Here is some R code to create the dataset:

```
Deaths = c(0,1,2,3,4,5)
N = c(109,65,22,3,1,0)
     # expand the dataset into individual cases
D1 = rep(Deaths,N)
```

   (a) An 'exact' confidence interval using the same idea as the Clopper-Pearson interval for the binomial, ie solve the following equations for lower and upper bounds, for

$$Y = \sum X_i \sim \text{Poisson}(n\theta):$$

$$\theta_L = \inf\{\theta : \mathbb{P}(Y \geq Y_{obs}|\theta) \geq .025\}$$
$$\theta_U = \sup\{\theta : \mathbb{P}(Y \leq Y_{obs}|\theta) \geq .025\}$$

   (b) The asymptotic confidence interval based using the fact that $\bar{X}$ is approximately Normal(θ,$1/I_n(\theta)$).

   (c) The confidence interval based on the asymptotic normality of the score function, that is, solve the following for (θ_L, θ_U):

$$\left| \frac{U_n(\tilde{X})}{\sqrt{I_n(\theta)}} \right| = 1.96$$

   (d) The confidence interval based on the likelihood ratio test statistic:

$$2(log(f(\tilde{X}|\hat{\theta})) - log(f(\tilde{X}|\theta))) \sim \chi^2_p$$

   In other words,

$$\{\theta : log(f(\tilde{X}|\hat{\theta})) - log(f(\tilde{X}|\theta))) \leq 3.84/2\}$$

   (e) A Bayesian HPD region of posterior probability .95 using a Gamma($5,\frac{1}{2}$) prior. Hint: the R function pgamma(x,a,b) computes the probability that a Gamma(a,b) random variable is less than x, that is the CDF $F(x|a,b)$. Hence the probability of the interval (X_L, X_U) is $F(X_U \mid a,b) - F(X_L \mid a,b)$.