Mathematics 361: Number Theory
Assignment E

Reading: Ireland and Rosen, Chapter 5 (including the exercises) and into Chapter 6

Problems:

1. (Don't turn this in, but make sure to do it if you haven't yet.) Confirm for odd primes p that

$$(-1)^{(p-1)/2} = \begin{cases} 1 & \text{if } p \equiv 1 \pmod 4, \\ -1 & \text{if } p \equiv 3 \pmod 4, \end{cases}$$

and

$$(-1)^{(p^2-1)/8} = \begin{cases} 1 & \text{if } p \equiv \pm 1 \pmod 8, \\ -1 & \text{if } p \equiv \pm 3 \pmod 8. \end{cases}$$

2. Use Quadratic Reciprocity to prove Euler's observation by justifying the following computation for distinct odd primes p and q with $4q > p$:

$$\begin{aligned}\left(\frac{q}{4q \pm p}\right) &= (-1)^{\frac{q-1}{2}\cdot\frac{4q\pm p-1}{2}}\left(\frac{4q \pm p}{q}\right) = (-1)^{\frac{q-1}{2}\cdot\frac{\pm p-1}{2}}\left(\frac{\pm p}{q}\right) \\ &= (-1)^{\frac{q-1}{2}\cdot\frac{p-1}{2}}\left(\frac{p}{q}\right) = \left(\frac{q}{p}\right).\end{aligned}$$

3. Read Ireland and Rosen Exercises 5.3; 5.4–5.8; 5.9, 5.10; 5.11; 5.12, 5.13; 5.14, 5.15; 5.16; 5.17 (just the Proposition); 5.18–5.21; 5.22; 5.23, 5.24; 5.38, and work a small selection of them.

4. Work Ireland and Rosen Exercises 5.25–5.28 and/or 5.29–5.31.