MATHEMATICS 332: ALGEBRA — ASSIGNMENT 5

Reading: Gallian, chapters 11, 24

Problems:

1. (*Gallian 11.9*) *Suppose that G is an abelian group of order 120 and that G has exactly three elements of order 2. Determine the isomorphism class of G.*

2. (*Gallian 11.10*) *Find all abelian groups of order 360.*

3. (*Gallian 11.19*) *The set*

$$\{1, 9, 16, 22, 29, 53, 74, 79, 81\}$$

is a subgroup of $(\mathbb{Z}/91\mathbb{Z})^{\times}$. Determine its isomorphism class.

4. (*Gallian 11.21*) *Characterize those positive integers n such that the only abelian groups of order n are cyclic.*

5. (*Gallian 24.11*) *Suppose that G is a group of order 168. If G has more than one 7-Sylow subgroup then how many 7-Sylow subgroups does it have?*

6. (*Gallian 24.14*) *Let G be a noncyclic group of order 21. How many 3-Sylow subgroups does G have?*

7. (*Gallian 24.18*) *Prove that a group of order 175 is abelian.*

8. (*Gallian 24.24*) *Prove that a group of order 105 contains a subgroup of order 35.*

9. (*Gallian 24.26*) *Let G be a group of order 60. Show that G has exactly four elements of order 5 or exactly 24 elements of order 5. Which of these cases holds for A_5?*

10. (*Gallian 24.38*) *Let G be a group of order $p^2 q^2$ where p and q are distinct primes such that $p \nmid q^2 - 1$ and $q \nmid p^2 - 1$. Prove that G is abelian.*