BASIC SYMBOL-PATTERNS FOR SETS AND MAPS

Let X and Y be sets, assuming no further structure whatsoever. Let

$$f : X \longrightarrow Y$$

be a function, with $Y = f(X)$ but subject to no other assumptions. Let I be any index set. Let

$$S, \quad \{S_i : i \in I\}$$

be arbitrary subsets of X, and let

$$T, \quad \{T_i : i \in I\}$$

be arbitrary subsets of Y. As usual, let f^{-1} denote inverse image, and let ^{c} denote complementation.

Each of the following eight pairs is plausibly related:

$$\begin{array}{ll}
f\left(\bigcup S_i\right) \text{ and } \bigcup f(S_i), & f^{-1}\left(\bigcup T_i\right) \text{ and } \bigcup f^{-1}(T_i), \\
f\left(\bigcap S_i\right) \text{ and } \bigcap f(S_i), & f^{-1}\left(\bigcap T_i\right) \text{ and } \bigcap f^{-1}(T_i), \\
f(S^c) \text{ and } (f(S))^c, & f^{-1}(T^c) \text{ and } (f^{-1}(T))^c, \\
f^{-1}(f(S)) \text{ and } S, & f(f^{-1}(T)) \text{ and } T.
\end{array}$$

For each pair, determine whether equality must hold, and if it doesn't, whether a containment must hold. When a containment holds, what conditions force it to be equality?