Northwest Undergraduate Mathematics Symposium

Reed College, Portland OR
April 9, 2011

Abstracts

Invited Address

Professor Naiomi Cameron, Lewis and Clark College
4:00–5:00, Vollum Lecture Hall

Combinatorial enumeration with the Riordan group

First described in 1991, the Riordan group is a mathematical structure which lies at the intersection of algebra and combinatorics. In essence, it is a special collection of infinite dimensional matrices whose entries are associated with combinatorial sequences. Because of the combinatorial nature of the entries in Riordan matrices, they can be used as an interest- ing tool for many counting problems. In fact, one of the most recognizable members of the Riordan group goes by the name Pascals Triangle. In this talk, I will provide a basic introduction to the Riordan group and present some counting problems related to trees and paths for which the Riordan group proves useful. The Fibonacci and Catalan numbers will be impor- tant characters in this story. Finally, I will describe some open questions related to the algebraic structure of the Riordan group.

Sessions

Matt Anderson, Willamette University
11:30–12:00, Psychology 105

Prime constellations

Patterns exist in the prime numbers. I will present the prime number theorem and also describe the Hardy Littlewood conjectures and depict some undiscovered patterns.

Sebastian Bozlee, University of Portland
10:15–10:30, Psychology 102/103

Behavior of alternating integrals

An “alternating integral” is defined and its basic properties are investigated, including closed form equations for polynomial functions, as well as addition, subtraction, and scalar multiplication formulas.

Rachel Burton and Jadon Herron, Eastern Oregon University
2:45–3:15, Psychology 102/103

Designing a halfpipe for advanced snurfers

The shape of a snowboarding halfpipe can be designed to maximize the vertical air a professional snowboarder will achieve; however, a practical course will take into account factors such as the number of cycles a snowboarder can make side to side as he descends the mountain, and the shape that yields the best launch for stunts.

As a snowboarder descends a halfpipe the forces acting on him are dependent on the shape of the course. We will use Newton's Second Law of Motion to sum the forces in each direction. Then we used a Runga-Kutta method computer simulation to determine the acceleration of the snowboarder down the course. We tested three different functions that describe the shape of the half pipe, with three varying slopes for each. Using a computer simulation and physical testing the velocity of the snowboarder can be analyzed at the edge of the half pipe. The shapes that yield the largest vertical air and the most practical course will then be chosen as the best design.

The forces taken into consideration in these models include: gravity, centrifugal, frictional, air resistance, and a constant force applied in the direction of travel that mimics the force from a professional snowboarder pumping his legs. These models do not take into account the force a professional snowboarder's stylistic force would use to guide his path along the halfpipe.

After running our three models we determined that out of the various shapes tested the best vertical height was optimized at a mountain slope of 45 degrees with a half pipe shape of the parabola $z(y) = .07y^2 - 7$.

The best practical model was found to be the same parabola at a mountain slope of 25 degrees. The Runge Kutta method approximation had an error of 0.1%. Our best model is the comprehensive three dimensional force because it most realistically describes a snowboarder on a halfpipe.

Liam Dalton, Pacific University
10:15–10:30, Psychology 105

Minimizing repeaters through probabalistic methods

Given the many limitations on the placement of radio technology in the telecommunications industry, it is often important to understand how terrain type, client density, and other factors influence the level of investment required when installing hardware to handle communications traffic. In this paper, a method for estimating the required technological investment for a region of clients is implemented and analyzed. This method was created in response to a 2011 MCM challenge.

Michael Donatz, Oregon State University
11:30–12:00, Psychology 102/103

Partitions, braids, and dynamical systems

This is an expository talk on some exploration done at OSU on properties of integer partitions (namely in relation with Andrew's Smallest part function), some ideas on representing partitions with braids, and some musings on directions to go with dynamical systems and their relation to partitions.

Thomas Eliot, Willamette University
1:30–1:45, Biology 19

An expansion of Euclid's proof of the uniqueness of the Platonic solids to work in every dimension

The climax of Euclid's Elements is his proof of the uniqueness of the Platonic Solids, the most symmetrical of all possible 3-dimensional objects. In this talk I will generalize the definition of a Platonic Solid to Convex Regular Polytopes, so it works in any dimension. I shall then create all of the CRPs in every dimension using Euclid's technique, along the way proving that these are the only possible ones.

Laura Florescu, Reed College
10:30–11:00, Psychology 102/103

Symmetric configurations in the Abelian sandpile model

This presentation will consist of a few theorems connecting the symmetric recurrent sandpile configurations on grid graphs to the dimmer model on grids and Möbius strips. Two main symmetries are investigated: Klein Four Group and dihedral.

Chris Harvey, University of Portland
2:45–3:00, Psychology 105

Methods for updating seasonal items with intermittent demand

Advanced forecasting methods are used in operations research to optimize product inventory levels. This study focuses on improving probability models replenishing inventory of items with intermittent and seasonal demand. Our models were programmed using R, but because the initial code implemented was not engineered for speed or efficient memory use, it limited our ability to quickly test different models. To address this problem the simulations were re-engineered in a manner that compartmentalized the code into distinct pieces that were unaware of the overarching data structure. This resulted in simulations with significantly decreased run times. The modular nature of the main simulation code then allowed for implementation of various updating methods. One such method involved updating model parameters with exponential smoothing. These updated parameters were then used to compare the results of different inventory reordering policies to see which performed the best for intermittent seasonal demand. Due to long run times, this would have been unthinkable with the original software architecture.

Duncan McGregor, Pacific University
3:00–3:15, Psychology 105

Combinatorial proofs of Zeckendorf representations

Arthur Benjamin proposed that a Fibonacci number was the number of ways to tile an $1 \times n$ board with 1×1 tiles and 1×2 dominoes. Eduard Zeckendorf proved that every natural number could be constructed as a sum of non-consecutive Fibonacci numbers. We call that sum the Zeckendorf Representation of a number. We prove several previously known Zeckendorf representations and produce new results.

Erik Hortsch, Oregon State University
10:15–10:30, Biology 19

Generalized polynomial chaos and dispersive dielectric media

We investigate polynomial chaos as a method to improve the accuracy of the Debye model and allow for easy simulation using the Finite Difference Time Domain (FDTD) method. We develop a framework for incorporating distributions of relaxation times using polynomial chaos and demonstrate that the resultant system adds no physical conditions. We conclude by demonstrating the exponential convergence of the system under a correct choice of polynomials and show how an inverse problem may be formulated to determine parameters of the distribution from experimental data or comparison with another model.

Kady Hossner, Western Oregon University
1:45–2:15, Biology 19

How to build Cayley-sudoku tables

In Abstract Algebra we learn that every Cayley Table (an operation table of a Group) is also a Latin Square. Here we will see what restrictions are needed to form "Cayley-Sudoku Tables." A Cayley-Sudoku Table is a Cayley Table but it also has smaller subsquares that contain every element of the Group exactly once.

David Krueger, Reed College
2:00–2:30, Psychology 105

The critical group of an oriented matroid

To every graph corresponds an abelian group known as the critical group. Oriented Matroids are structures that encode a type of generalizes linear independence. Using graphic matroids (constructed from the cycles or cuts of a graph) as an inspiration, we define the critical group of an oriented matroid and give some basic results.

Cynthia Lester, Linfield College
11:00–11:30, Psychology 102/103

Tilings with T and skew tetrominoes

The last ten years have brought some interesting new methods and results in questions involving tiling regions in the integer lattice. In this context we solve a new tiling question involving T-tetrominoes, skew tetrominoes, and modified rectangles. Coloring arguments are the main method for proving un-tileability of these regions.

Aubrey Leung, Oregon State University
10:30–11:00, Biology 19

A sequential operator splitting method for Maxwell's equations in Debye dispersive media

Operator splitting methods are a family of numerical methods by which a complex or in some respect difficult operator is split apart into suboperators, which can be individually solved and then recombined. We discuss applications of sequential operator splitting techniques to Maxwell's equations for electromagnetic waves in a one dimensional Debye medium. We compare this method to the Yee scheme and conduct accuracy, stability, and dispersion analysis on the operator splitting scheme.

Rosie Leung, Oregon State University
2:30–2:45, Psychology 105

The effect of localized oil spills on the Atlantic loggerhead turtle population dynamics

The loggerhead sea turtle (*Caretta caretta*) is an endangered species with three regional nesting populations in the Gulf of Mexico and the western North Atlantic Ocean. We develop a spatial, stage-classified matrix model of these populations and use it to investigate the effects of localized oil spills in each nesting region. We examine oil-induced mortality ranging from 25% to 100% and affecting stage classes either proportionally or equally. The results of this study are intended to provide insights into the population dynamics of the Atlantic loggerhead turtles and suggest conservation techniques appropriate in each oil spill case.

Allison Lewis, University of Portland
11:30–12:00, Biology 19

Theory and applications of Benford's Law

Benfords Law of Leading Digits has contributed to the analysis of a variety of real-life data sets and provides us with a way to detect abnormalities in data resulting from rounding errors, data collection methods, or even fraud and other nefarious activities. This study applies Benford tests to several diverse data sets, including data from the controversial 2009 Iranian election and a portion of the data from the Climategate scandal, in an effort to analyze conformity of each of these data sets to the expected distributions as outlined by Benfords Law. We also explore the theoretical implications of Benfords Law, analyzing various Weibull distributions and determining how varying the parameters affects their conformity to the expected leading digit probabilities. The major goal of this study is to determine which data sets should be governed by this law, based on factors such as size and the presence of data entries spanning multiple orders of magnitude.

Dan Lidral-Porter, Reed College
11:00–11:30, Biology 19

Compound sorting networks

Sorting networks are a well-known model for parallel sorting algorithms, wherein a fixed network of lines and comparators produces a sorted output for all possible inputs. One of their advantages is ease of implementation in BSP or message-passing models of parallel computation, yet their restriction that each line contain only one value is impractical. This talk will provide background on sorting networks, and then present an extension of networks to compound networks, where each line contains multiple values. We'll prove that many compound versions of sorting networks retain their sorting property, and see that their asymptotic complexity and real-world performance improves. We will also look at some cases where the sorting property breaks down, and consider strategies for modifying such networks back into sorting networks.

Andrea Olson, Western Oregon University
1:30–2:00, Psychology 102/103

The math of magic

In 1593, Horatio Galasso published Giochi di carte bellissimi di regola e di memoria, a book on various magic tricks. This was the first book published explaining the 21 card trick, a trick in which the magician, after various movements of cards, discovers the volunteers previously chosen card. I will be analyzing the trick and how it works, as well as analyzing an extension.

Julia Porcino, Reed College
2:45–3:15, Biology 19

Analysis of permanental ideals over hypermatrices

Permanental ideals are ideals generated by permanents taken over all 2×2 submatrices of a given matrix. In a 2000 paper, Laubenbacher and Swanson analyzed these permanental ideals over all 2-dimensional matrices. In this talk, I will present results from my thesis, which analyzes permanental ideals over 3-dimensional hypermatrices. I will provide background on permanents, permanental ideals and their relations to determinantal ideals, and then share results pertaining to the minimal primes over three different types of permanental ideals.

Siddharth Raval, Reed College
2:30–2:45, Psychology 102/103

Threshold dynamical systems

Many social contagions require multiple contacts or sources of activation to spread. Examples include adoption of new technologies, participation in social movements, etc. Such processes are spread by 'complex contagions'. So far, these have been modeled by threshold systems with fixed thresholds.

However, a fixed threshold does not capture the dynamics of systems involving, for example, varying disease immunity, evolving psychological factors in social contagions, etc. We generalize the notion of threshold systems to incorporate these dynamics. Namely, we develop increasing, decreasing, and mixed systems that better reflect realistic dynamics; we call this generalization an evolving threshold dynamical system.

Using the framework of graph dynamical systems, we characterize the long term behavior of evolving threshold systems, enumerate the limit sets, describe the phase space of such systems and, in the process, elucidate topological conjugacies between maps of these system's threshold functions. We then consider ET-DSs on some specific graph classes, deriving explicit formulae for the number of fixed points.

Nick Salter, Reed College
1:30–2:00, Psychology 105

Fourier-analytic methods in polytope lattice-point enumeration

A central theme in the study of polytopes is the interplay between the associated discrete and continuous data. One particularly simple example of this is the Euler-Maclaurin summation formula, which expresses the sum of a function $f : \mathbb{R} \to \mathbb{C}$ at the integer points of an interval in terms of the continuous data encoded in the integral of f over the interval. Interpolating between the sums and the integrals are the Bernoulli polynomials, objects of great number-theoretic significance.

Viewing an interval on the real line as a one-dimensional polytope provides a starting point for developing a polytope Euler-Maclaurin formula in arbitrary dimension. In this talk I will present work from my senior thesis which realizes such a formula as an application of the Poisson summation formula from Fourier analysis. Time permitting, I will also discuss the multivariate Bernoulli-like objects that arise from this process.

Tim Sasaki, Western Oregon University
2:00–2:30, Psychology 102/103

Combination locks and permutations

Suppose that we have a combination lock with buttons numbered 1 to n, where combinations are constructed by pressing each button on the lock precisely one time. A natural way to think of possible combinations would be to consider all of the ones possible when buttons on the lock are pushed one at a time. This clearly results in $n!$ possible combinations. However, if we allow for multiple buttons to be pushed simultaneously, what effect does this have on the number of combinations? The resulting set of lock combinations ends up having connections to both the Eulerean numbers and Stirling numbers of the second kind.

Mary Solbrig, Reed College
2:15–2:45, Biology 19

Frobenius groups and representation induction

A basic introduction to group representation theory and the representations of Frobenius Groups. This will cover the definition of a representation, reducibility, how to create a group representation from a representation of a subgroup, and the characters of Frobenius Groups. Aimed at an undergraduate with a basic algebra background.

Gaurav Venkataraman, Reed College
10:30–11:00, Psychology 105

A few results on the interaction of spike train metrics with model neurons

It is well known that neurons 'fire.' Over the time course of an experiment or action, these series of neuronal firings is called a spike train. A fundamental issue in theoretical neuroscience is how to quantify dissimilarity between spike trains, thereby illuminating the temporal coding properties therein. We will consider the spike train metric approach to this problem, where spike trains are thought of as points in a metric space, where the particular metric is defined to meet some theoretical or physiological need. In particular, we will focus on the way that spike train metrics interact with various types of model neurons, and present newly defined metrics to solve various theoretical problems. The work itself is a blend of topology, biology, statistics, and computation.

Tianyuan Xu, Reed College 11:00–11:30, Psychology 105

Graph partitions and minimal free resolutions

We study relations (syzygies) among polynomial equations associated with the Laplacian of a finite graph. The talk will show how the connected partitions of a graph illuminate important ideas in the fields of commutative algebra and algebraic geometry.