HW 6, due Friday, March 15 *Math 411*

1. On $\mathbb{P}^1$, we have the orienting atlas $\mathfrak{A} = \{(U_x, h), (U_y, k)\}$ where

$$\begin{aligned} h\colon U_x := \{(x,y) \in \mathbb{P}^1 : x \neq 0\} &\to \mathbb{R} \\ (x,y) &\mapsto y/x \end{aligned}$$

$$\begin{aligned} k\colon U_y := \{(x,y) \in \mathbb{P}^1 : y \neq 0\} &\to \mathbb{R} \\ (x,y) &\mapsto -x/y. \end{aligned}$$

Define $\omega \in \Omega^1\mathbb{P}^1$ whose local description with respect to h is

$$\omega|_{U_x}(a) = \frac{1}{a^2+1}\, da.$$

In HW 4, we saw that this completely determines ω on $\mathbb{P}^1$.

(a) Let $A_1 = h^{-1}([-1,1])$ and $A_2 = k^{-1}([-1,1])$. Show that $A_1 \cup A_2 = \mathbb{P}^1$ with intersection of measure 0.

(b) Compute $\int_{\mathbb{P}^1} \omega$ using A_1 and A_2 (following the definition of the integral given in class).

2. Let S^1 be the unit circle in $\mathbb{R}^2$ centered at the origin. Define $p(t) = (\cos(t), \sin(t))$ and $q(t) = p(t+\pi)$ for $t \in (-\pi, \pi)$. Use p^{-1} and q^{-1} as local coordinates on S^1.

(a) Compute the transition function $q^{-1} \circ p$.

(b) In the local coordinate t provided by p, consider $dt_{(1,0)} \in T^*_{(1,0)}S^1$. For each angle α there is a mapping $m_\alpha\colon S^1 \to S^1$ sending a point with angle β to the point with angle $\alpha + \beta$. Define

$$\omega(\alpha) = m^*_\alpha(dt_{(1,0)}).$$

What is the 1-form ω in the local coordinate provided by p?

(c) What is $\int_{S^1} \omega$?

3. Read the handout on the homology of simplicial complexes. Turn in problems 1, 2, 3, and 5.