Errata
Instructor's Solutions Manual
Introduction to Quantum Mechanics
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- Page 3, Prob. 1.6(b): last two lines should read

$$= A\left[\frac{1}{2\lambda}\sqrt{\frac{\pi}{\lambda}} + 0 + a^2\sqrt{\frac{\pi}{\lambda}}\right] = \boxed{a^2 + \frac{1}{2\lambda}.}$$

$$\sigma^2 = \langle x^2\rangle - \langle x\rangle^2 = a^2 + \frac{1}{2\lambda} - a^2 = \frac{1}{2\lambda}; \ \boxed{\sigma = \frac{1}{\sqrt{2\lambda}}.}$$

- Page 8, Prob. 2.6(b): in the first box, the argument of the second sine should include an x.
- Page 9, Prob. 2.9: in the last line, $c_1 = 8\sqrt{15}/\pi^3$.
- Page 10, Prob. 2.11: in the third line the proof assumes that g (which in our case will be $a_-\psi$) does not actually *blow up* at $\pm\infty$ faster than f (in our case ψ) goes to zero. I don't know how to fix this defect without appealing to the analytic approach, where we find (Eq. 2.60) that $\psi(x)$ goes asymptotically like $e^{-m\omega x^2/2\hbar}$, and hence so too does $a_-\psi$.
- Page 10, Prob. 2.12: Because of the i and $-i$ inserted in Eqs. 2.52 and 2.53 respectively (see Corrections #2—June 1997), the expression for c at the end of (a) should include a factor of i. Also, add at the end of (a): "(The signs are conventional.)" In part (b), every $\sqrt{\hbar\omega}$ should carry a factor of i (i.e. insert i three times in the first line, i^n three times in the next line, and $(-i)^n$ in the boxed answer).
- Page 11, Prob. 2.14(a): for the same reason, in the third line, remove the i in the expression for ψ_1.
- Page 19, Prob. 2.30: or "$\ldots \tan z \approx z = \sqrt{(z_0/z)^2 - 1} = (1/z)\sqrt{z_0^2 - z^2}$. Now (Eqs. 2.130 and 2.137) $z_0^2 - z^2 = \kappa^2 a^2$, so $z^2 = \kappa a$. But $z_0^2 - z^2 = z^4 \ll 1 \Rightarrow z \approx z_0$, so $\kappa a \approx z_0^2 \ldots$ "
- Page 22, Prob. 2.36: remove box, and continue as follows:

$$\Psi(x,t) = \frac{1}{\sqrt{10}}\left[3\psi_1(x)e^{-iE_1t/\hbar} - \psi_3(x)e^{-iE_3t/\hbar}\right],$$

$$|\Psi(x,t)|^2 = \frac{1}{10}\left[9\psi_1^2 + \psi_3^2 - 6\psi_1\psi_3\cos\left(\frac{E_3 - E_1}{\hbar}t\right)\right],$$

so

$$\begin{aligned}\langle x\rangle &= \int_0^a x|\Psi(x,t)|^2\,dx\\ &= \frac{9}{10}\langle x\rangle_1+\frac{1}{10}\langle x\rangle_3-\frac{3}{5}\cos\left(\frac{E_3-E_1}{\hbar}t\right)\int_0^a x\psi_1(x)\psi_3(x)\,dx,\end{aligned}$$

where $\langle x\rangle_n = a/2$ is the expectation value of x in the nth stationary state. The remaining integral is

$$\frac{2}{a}\int_0^a x\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{3\pi x}{a}\right)\,dx = \frac{1}{a}\int_0^a x\left[\cos\left(\frac{2\pi x}{a}\right)-\cos\left(\frac{4\pi x}{a}\right)\right]dx$$

$$= \frac{1}{a}\left[\left(\frac{a}{2\pi}\right)^2\cos\left(\frac{2\pi x}{a}\right)+\left(\frac{xa}{2\pi}\right)\sin\left(\frac{2\pi x}{a}\right)-\left(\frac{a}{4\pi}\right)^2\cos\left(\frac{4\pi x}{a}\right)\right.$$
$$\left.-\left(\frac{xa}{4\pi}\right)\sin\left(\frac{4\pi x}{a}\right)\right]\Bigg|_0^a = 0.$$

Evidently, then,

$$\langle x\rangle = \frac{9}{10}\left(\frac{a}{2}\right)+\frac{1}{10}\left(\frac{a}{2}\right) = \boxed{\frac{a}{2}.}$$

- Page 23, Prob. 2.37: Because of the i and $-i$ inserted in Eqs. 2.52 and 2.53 respectively (see Corrections #2—June 1997), line 3 on should read as follows:

$$\langle x\rangle = \frac{-i}{\omega\sqrt{2m}}\int \psi_n^*(a_+-a_-)\psi_n\,dx.$$

But $\left\{\begin{array}{lcll} a_+\psi_n &=& i\sqrt{(n+1)\hbar\omega}\psi_{n+1} & [2.52]\\ a_-\psi_n &=& -i\sqrt{n\hbar\omega}\psi_{n-1} & [2.53]\end{array}\right\}$, so

$$\langle x\rangle = \frac{1}{\omega\sqrt{2m}}\left[\sqrt{(n+1)\hbar\omega}\int\psi_n^*\psi_{n+1}\,dx+\sqrt{n\hbar\omega}\int\psi_n^*\psi_{n-1}\,dx\right] = \boxed{0}$$

(by the orthogonality of $\{\psi_n\}$). Also $\langle p\rangle = m\dfrac{d\langle x\rangle}{dt} = \boxed{0.}$ Meanwhile

$$\hat{x}^2 = \frac{-1}{2m\omega^2}(a_+-a_-)(a_+-a_-) = \frac{-1}{2m\omega^2}(a_+^2-a_+a_--a_-a_++a_-^2),$$

so $\langle x^2\rangle = \dfrac{-1}{2m\omega^2}\displaystyle\int\psi_n^*(a_+^2-a_+a_--a_-a_++a_-^2)\psi_n\,dx.$ But

$$\left\{\begin{array}{lclcl} a_+^2\psi_n &=& a_+(i\sqrt{(n+1)\hbar\omega}\psi_{n+1}) &=& -\sqrt{(n+1)(n+2)}\hbar\omega\psi_{n+2}.\\ a_+a_-\psi_n &=& a_+(-i\sqrt{n\hbar\omega}\psi_{n-1}) &=& n\hbar\omega\psi_n.\\ a_-a_+\psi_n &=& a_-(i\sqrt{(n+1)\hbar\omega}\psi_{n+1}) &=& (n+1)\hbar\omega\psi_n.\\ a_-^2\psi_n &=& a_-(-i\sqrt{n\hbar\omega}\psi_{n-1}) &=& -\sqrt{n(n-1)}\hbar\omega\psi_{n-2}.\end{array}\right.$$

[The rest is unchanged.]

- Page 32, Prob. 3.2(b): in the odd case the dimension is $(N+1)/2$.
- Pages 49-50, Prob. 3.50: Because of the i and $-i$ inserted in Eqs. 2.52 and 2.53 respectively (see Corrections #2—June 1997), the solution should be changed to read as follows:

$$x = -\frac{i}{\omega\sqrt{2m}}(a_+ - a_-) \quad \text{[Prob. 2.37].} \begin{cases} a_+|n\rangle = i\sqrt{(n+1)\hbar\omega}|n+1\rangle, \\ a_-|n\rangle = -i\sqrt{n\hbar\omega}|n-1\rangle. \end{cases}$$

$$\begin{aligned} \langle n|x|n'\rangle &= \tfrac{-i}{\omega\sqrt{2m}}\langle n|(a_+ - a_-)|n'\rangle \\ &= \tfrac{-i}{\omega\sqrt{2m}}\left[i\sqrt{(n'+1)\hbar\omega}\langle n|n'+1\rangle + i\sqrt{n'\hbar\omega}\langle n|n'-1\rangle\right] \\ &= \sqrt{\tfrac{\hbar}{2m\omega}}\left(\sqrt{n'+1}\delta_{n,n'+1} + \sqrt{n'}\delta_{n,n'-1}\right) \\ &= \boxed{\sqrt{\frac{\hbar}{2m\omega}}\left(\sqrt{n}\delta_{n',n-1} + \sqrt{n'}\delta_{n,n'-1}\right).} \end{aligned}$$

$$p = \sqrt{\frac{m}{2}}(a_+ + a_-) \Rightarrow \langle n|p|n'\rangle = \boxed{i\sqrt{\frac{m\hbar\omega}{2}}\left(\sqrt{n}\delta_{n',n-1} - \sqrt{n'}\delta_{n,n'-1}\right).}$$

Noting that n and n' run from *zero* to infinity, the matrices are:

$$\boxed{X = \sqrt{\frac{\hbar}{2m\omega}}\begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & 0 & 0 & \\ \sqrt{1} & 0 & \sqrt{2} & 0 & 0 & 0 & \\ 0 & \sqrt{2} & 0 & \sqrt{3} & 0 & 0 & \vdots \\ 0 & 0 & \sqrt{3} & 0 & \sqrt{4} & 0 & \\ 0 & 0 & 0 & \sqrt{4} & 0 & \sqrt{5} & \\ & & & \cdots & & & \end{pmatrix}}$$

$$\boxed{P = i\sqrt{\frac{m\hbar\omega}{2}}\begin{pmatrix} 0 & -\sqrt{1} & 0 & 0 & 0 & 0 & \\ \sqrt{1} & 0 & -\sqrt{2} & 0 & 0 & 0 & \\ 0 & \sqrt{2} & 0 & -\sqrt{3} & 0 & 0 & \vdots \\ 0 & 0 & \sqrt{3} & 0 & -\sqrt{4} & 0 & \\ 0 & 0 & 0 & \sqrt{4} & 0 & -\sqrt{5} & \\ & & & \cdots & & & \end{pmatrix}}$$

Squaring these matrices:

$$X^2 = \frac{\hbar}{2m\omega}\begin{pmatrix} 1 & 0 & \sqrt{1\cdot 2} & 0 & 0 & 0 & \\ 0 & 3 & 0 & \sqrt{2\cdot 3} & 0 & 0 & \\ \sqrt{1\cdot 2} & 0 & 5 & 0 & \sqrt{3\cdot 4} & 0 & \vdots \\ 0 & \sqrt{2\cdot 3} & 0 & 7 & 0 & \sqrt{4\cdot 5} & \\ & & & \cdots & & & \end{pmatrix};$$

$$P^2 = -\frac{m\hbar\omega}{2}\begin{pmatrix} -1 & 0 & \sqrt{1\cdot 2} & 0 & 0 & 0 & \\ 0 & -3 & 0 & \sqrt{2\cdot 3} & 0 & 0 & \\ \sqrt{1\cdot 2} & 0 & -5 & 0 & \sqrt{3\cdot 4} & 0 & \vdots \\ 0 & \sqrt{2\cdot 3} & 0 & -7 & 0 & \sqrt{4\cdot 5} & \\ & & & \cdots & & & \end{pmatrix}.$$

So the Hamiltonian, in matrix form, is

$$\begin{aligned} H &= \frac{1}{2m}P^2 + \frac{m\omega^2}{2}X^2 \\ &= -\frac{\hbar\omega}{4}\begin{pmatrix} -1 & 0 & \sqrt{1\cdot 2} & 0 & 0 & 0 & \\ 0 & -3 & 0 & \sqrt{2\cdot 3} & 0 & 0 & \\ \sqrt{1\cdot 2} & 0 & -5 & 0 & \sqrt{3\cdot 4} & 0 & \vdots \\ 0 & \sqrt{2\cdot 3} & 0 & -7 & 0 & \sqrt{4\cdot 5} & \\ & & & \cdots & & & \end{pmatrix} \\ &\quad +\frac{\hbar\omega}{4}\begin{pmatrix} 1 & 0 & \sqrt{1\cdot 2} & 0 & 0 & 0 & \\ 0 & 3 & 0 & \sqrt{2\cdot 3} & 0 & 0 & \\ \sqrt{1\cdot 2} & 0 & 5 & 0 & \sqrt{3\cdot 4} & 0 & \vdots \\ 0 & \sqrt{2\cdot 3} & 0 & 7 & 0 & \sqrt{4\cdot 5} & \\ & & & \cdots & & & \end{pmatrix} \\ &= \frac{\hbar\omega}{2}\begin{pmatrix} 1 & 0 & 0 & 0 & \\ 0 & 3 & 0 & 0 & \\ 0 & 0 & 5 & 0 & \\ 0 & 0 & 0 & 7 & \\ & & & & \ddots \end{pmatrix}. \end{aligned}$$

The diagonal elements are $H_{nn} = (n + 1/2)\hbar\omega$, as they should be.

- Page 53, Prob. 3.57(a): add the following:

  If $|\gamma\rangle$ is an eigenvector of $\hat{P}$ with eigenvalue λ, then $\hat{P}|\gamma\rangle = \lambda|\gamma\rangle$, and it follows that $\hat{P}^2|\gamma\rangle = \lambda\hat{P}|\gamma\rangle = \lambda^2|\gamma\rangle$. But $\hat{P}^2 = \hat{P}$, and $|\gamma\rangle \neq 0$, so $\lambda^2 = \lambda$, and hence the eigenvalues of $\hat{P}$ are $\boxed{\text{0 and 1.}}$ Any (complex) *multiple* of $|\alpha\rangle$ is an eigenvector of $\hat{P}$, with eigenvalue 1; any vector *orthogonal* to $|\alpha\rangle$ is an eigenvector of $\hat{P}$, with eigenvalue 0.

- Page 84, Prob. 5.6(b): all three minus signs should be plus.

- Page 87, Prob. 5.12: for the configuration given $S = 2$, so we need four unpaired electrons. In the proposed "likely" arrangement of the 30 extra electrons all shells are filled except 2 in the $4f$ state, so this doesn't work. The most probable arrangement is actually

$$(4d)^{10}(5s)^2(5p)^6(4f)^{10}(6s)^2.$$

- Page 88, Prob. 5.15(b): change to read "In this case [5.63] $\Rightarrow$ 0=0, [5.61] holds automatically, and [5.62] gives $ka - (\pm 1)k[A(\pm 1) - 0] = -(2m\alpha/\hbar^2)B \ \Rightarrow B = 0$." [The rest is unchanged.]

- Page 99, Prob. 6.4(b): in the last line, change "Problem 15(a)" to "Problem 6.2(a)".

- Pages 101-102, Prob. 6.7(b and c): starting with the fourth line of (b), all exponentials of the form

$$e^{-(2\pi n/L)^2 a} \qquad \text{should read} \qquad e^{-(2\pi n a/L)^2}.$$

  By my count this happens a total of six times.

- Page 106, Prob. 6.18: using $r = a$ is reasonable, since all we're looking for is a rough estimate, but the magnetic field in the ground state is problematic, so you might prefer to use, say, $n = 2$, $l = 1$; in that case (see Eq. 6.63) the answer is reduced by a factor of 24, to 0.5 T.

- Page 116-117, Prob. 6.31: in later printings the statement of the problem has been corrected, removing the minus signs in the displayed equation, and inserting one in the *Partial Answer*. The solution will match the revised version if the sign of E_{ext} is systematically reversed.

- Pages 121-124, Prob. 6.34: for some reason the solution, which begins on page 121, skips then to page 124, and finishes on page 123.

- Page 145, Prob. 9.1: top line, Table 4.5 should be Table 4.6; second line, the exponent in ψ_{210} should be $-r/2a$.

- Page 146, Prob. 9.3: This is a tricky problem, and I thank Prof. Onuttom Narayan for showing me the correct solution. The safest approach is to represent the delta function as a sequence of rectangles:

$$\delta_\epsilon(t) = \left\{ \begin{array}{ll} (1/2\epsilon), & -\epsilon < t < \epsilon, \\ 0, & \text{otherwise.} \end{array} \right\}$$

  We may as well set $t_0 = 0$ (in later printings this is done in the text); then

$$\left\{ \begin{array}{ll} t < -\epsilon: & c_a(t) = 1, \ c_b(t) = 0, \\ t > \epsilon: & c_a(t) = a, \ c_b(t) = b, \\ -\epsilon < t < \epsilon: & \left\{ \begin{array}{lcl} \dot{c}_a & = & -\frac{i\alpha}{2\epsilon\hbar} e^{-i\omega_0 t} c_b, \\ \dot{c}_b & = & -\frac{i\alpha^*}{2\epsilon\hbar} e^{i\omega_0 t} c_a. \end{array} \right. \end{array} \right\}$$

In the interval $-\epsilon < t < \epsilon$,

$$\begin{aligned}\frac{d^2c_b}{dt^2} &= -\frac{i\alpha^*}{2\epsilon\hbar}\left[i\omega_0 e^{i\omega_0 t}c_a + e^{i\omega_0 t}\left(\frac{-i\alpha}{2\epsilon\hbar}e^{-i\omega_0 t}c_b\right)\right] \\ &= -\frac{i\alpha^*}{2\epsilon\hbar}\left[i\omega_0\frac{i2\epsilon\hbar}{\alpha^*}\frac{dc_b}{dt} - \frac{i\alpha}{2\epsilon\hbar}c_b\right] = i\omega_0\frac{dc_b}{dt} - \frac{|\alpha|^2}{(2\epsilon\hbar)^2}c_b.\end{aligned}$$

Thus c_b satisfies a homogeneous linear differential equation with constant coefficients:

$$\frac{d^2c_b}{dt^2} - i\omega_0\frac{dc_b}{dt} + \frac{|\alpha|^2}{(2\epsilon\hbar)^2}c_b = 0.$$

Try a solution of the form $c_b(t) = e^{\lambda t}$:

$$\lambda^2 - i\omega_0\lambda + \frac{|\alpha|^2}{(2\epsilon\hbar)^2} = 0 \Rightarrow \lambda = \frac{i\omega_0 \pm \sqrt{-\omega_0^2 - |\alpha|^2/(\epsilon\hbar)^2}}{2},$$

or

$$\lambda = \frac{i\omega_0}{2} \pm \frac{i\omega}{2}, \text{ where } \omega \equiv \sqrt{\omega_0^2 + |\alpha|^2/(\epsilon\hbar)^2}.$$

The general solution is

$$c_b(t) = e^{i\omega_0 t/2}\left(Ae^{i\omega t/2} + Be^{-i\omega t/2}\right).$$

But

$$c_b(-\epsilon) = 0 \Rightarrow Ae^{-i\omega\epsilon/2} + Be^{i\omega\epsilon/2} = 0 \Rightarrow B = -Ae^{-i\omega\epsilon},$$

so

$$c_b(t) = Ae^{i\omega_0 t/2}\left(e^{i\omega t/2} - e^{-i\omega(\epsilon + t/2)}\right).$$

Meanwhile

$$\begin{aligned}c_a(t) &= \frac{2i\epsilon\hbar}{\alpha^*}e^{-i\omega_0 t}\dot{c}_b \\ &= \frac{2i\epsilon\hbar}{\alpha^*}e^{-i\omega_0 t/2}A\left[\frac{i\omega_0}{2}\left(e^{i\omega t/2} - e^{-i\omega(\epsilon+t/2)}\right) + \frac{i\omega}{2}\left(e^{i\omega t/2} + e^{-i\omega(\epsilon+t/2)}\right)\right] \\ &= -\frac{\epsilon\hbar}{\alpha^*}e^{-i\omega_0 t/2}A\left[(\omega+\omega_0)e^{i\omega t/2} + (\omega-\omega_0)e^{-i\omega(\epsilon+t/2)}\right].\end{aligned}$$

But

$$c_a(-\epsilon) = 1 = -\frac{\epsilon\hbar}{\alpha^*}e^{i(\omega_0-\omega)\epsilon/2}A\left[(\omega+\omega_0) + (\omega-\omega_0)\right] = -\frac{2\epsilon\hbar\omega}{\alpha^*}e^{i(\omega_0-\omega)\epsilon/2}A,$$

so $A = -\frac{\alpha^*}{2\epsilon\hbar\omega} e^{i(\omega-\omega_0)\epsilon/2}$. Therefore

$$\begin{aligned}
c_a(t) &= \frac{1}{2\omega} e^{-i\omega_0(t+\epsilon)/2} \left[(\omega+\omega_0) e^{i\omega(t+\epsilon)/2} + (\omega-\omega_0) e^{-i\omega(t+\epsilon)/2} \right] \\
&= e^{-i\omega_0(t+\epsilon)/2} \left\{ \cos\left[\frac{\omega(t+\epsilon)}{2}\right] + i\frac{\omega_0}{\omega} \sin\left[\frac{\omega(t+\epsilon)}{2}\right] \right\}; \\
c_b(t) &= -\frac{i\alpha^*}{2\epsilon\hbar\omega} e^{i\omega_0(t-\epsilon)/2} \left[e^{i\omega(t+\epsilon)/2} - e^{-i\omega(t+\epsilon)/2} \right] \\
&= -\frac{i\alpha^*}{\epsilon\hbar\omega} e^{i\omega_0(t-\epsilon)/2} \sin\left[\frac{\omega(t+\epsilon)}{2}\right].
\end{aligned}$$

Thus

$$\begin{aligned}
a &= c_a(\epsilon) = e^{-i\omega_0\epsilon} \left[\cos(\omega\epsilon) + i\frac{\omega_0}{\omega} \sin(\omega\epsilon) \right] \\
b &= c_b(\epsilon) = -\frac{i\alpha^*}{\epsilon\hbar\omega} \sin(\omega\epsilon).
\end{aligned}$$

This is for the rectangular pulse; it remains to take the limit $\epsilon \to 0$: $\omega \to |\alpha|/\epsilon\hbar$, so

$$\begin{aligned}
a &\to \cos\left(\frac{|\alpha|}{\hbar}\right) + i\frac{\omega_0\epsilon\hbar}{|\alpha|} \sin\left(\frac{|\alpha|}{\hbar}\right) \to \cos\left(\frac{|\alpha|}{\hbar}\right), \\
b &\to -\frac{i\alpha^*}{|\alpha|} \sin\left(\frac{|\alpha|}{\hbar}\right),
\end{aligned}$$

and we conclude that for the delta function

$$\boxed{\begin{aligned}
c_a(t) &= \begin{cases} 1, & t < 0, \\ \cos(|\alpha|/\hbar), & t > 0; \end{cases} \\
c_b(t) &= \begin{cases} 0, & t < 0, \\ -i\sqrt{\frac{\alpha^*}{\alpha}} \sin(|\alpha|/\hbar), & t > 0. \end{cases}
\end{aligned}}$$

Obviously, $|c_a(t)|^2 + |c_b(t)|^2 = 1$ in both time periods. Finally,

$$\boxed{P_{a\to b} = |b|^2 = \sin^2(|\alpha|/\hbar).}$$

- Page 168, Prob, 11.2: the one-dimensional case should read

$$\psi(x) \approx A\left\{ e^{ikx} + f(\text{sign}(x)) e^{ik|x|} \right\}.$$