Final Exam

Quantum Mechanics I
Physics 342

Date: May 12th, 2008

NAME:

Summer address (if you want your exam sent)

There are six problems, each worth 10 points, the examination will be graded out of 50. Show as much work as you can for all problems. You have three hours, and can use the front and back of a 3×5 (inch) card – the front and back covers of the book are provided separately.

Problem 1.1

Problems of a general mathematical nature.

a. What is the Fourier transform of the function $\delta(x-a)$?

b. Find the three independent solutions to the ordinary differential equation:

$$\frac{d^3 f(x)}{dx^3} = -\frac{df(x)}{dx} \tag{1}$$

c. Find, correct to two decimal places after the zero, the value of

$$\sqrt{101} \tag{2}$$

(no calculators allowed – Hint: Try perturbation).

Problem 1.2

Problems of a Quantum Mechanical nature.

a. Show that for the Hamiltonian:

$$H = a_+ a_- + \frac{1}{2}\,\hbar\,\omega, \tag{3}$$

with $[a_-, a_+] = 1$, if $|\psi_n\rangle$ is an eigenstate of H, then so is $a_+\,|\psi_n\rangle$.

b. Find the commutator of $\mathbf{L}$ and $\mathbf{r}$. Using this, write the generalized uncertainty bound for $\sigma_x^2\,\sigma_{L_y}^2$ – i.e. what is the right-hand side below:

$$\sigma_x^2\,\sigma_{L_y}^2 \geq \text{??} \tag{4}$$

Problem 1.3

For three spin $\frac{1}{2}$ particles, we can define the total spin:

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3. \tag{5}$$

a. Find the combination of the three spins corresponding to the eigenstate of S^2 with eigenvalue $\frac{15}{4}\,\hbar^2$ that is simultaneously an eigenstate of S_z with eigenvalue $\frac{1}{2}\,\hbar$.

b. Check your result using the Clebsch-Gordon table (Table 4.8 will be provided).

Problem 1.4

Find the energy spectrum of a particle under the influence of the one-dimensional Schrödinger equation for the harmonic oscillator potential with a shift: $V(x) = \frac{1}{2}\, m\, \omega^2\, x^2 + V_0$ with $V_0 > 0$.

Problem 1.5

We have an infinite square well with a (gently) sloped bottom:

$$V(x) = \begin{cases} \epsilon\, x & 0 < x < a \\ \infty & x < 0 \text{ and } x > a \end{cases} \tag{6}$$

Find the first order (for small ϵ) corrections to the energy spectrum of the infinite square well associated with this perturbation.

Problem 1.6

The following questions refer to three states of Hydrogen: $\psi_{2\,1\,-1}(r,\theta,\phi)$, $\psi_{2\,1\,0}(r,\theta,\phi)$ and $\psi_{2\,0\,0}(r,\theta,\phi)$.

a. Find the energy of each state (assume this is "classical" quantum mechanics – so we are using the Schrödinger equation with the Coulomb potential).

b. Sketch the radial dependence of each of the three wave functions below (use Table 4.6 if necessary).

c. Suppose we induce a transition from $\psi_{3\,2\,-2}(r, \theta, \phi)$ to each of the three states – calculate the frequency of light emitted for each of these transitions.

d. We are told that a system is initially in an equal admixture of the three given states (assume each is individually normalized) – what is $\Psi(r, \theta, \phi, t)$ (leave your answer in terms of $\psi_{2\,1\,-1}$, $\psi_{2\,1\,0}$ and $\psi_{2\,0\,0}$)?