

XII. Atomic Hydrogen and Deuterium Spectra: The Mass of the Deuteron

Discussion

The energy levels of a hydrogenic atom are given by¹

$$E_n = -\frac{1}{2} Z^2 \alpha^2 \mu c^2 \frac{1}{n^2} \quad (1)$$

where Z is the nuclear charge ($Z = 1$ for H and D atoms), α is the fine structure constant, μ is the reduced mass $\mu \equiv m_e m_N / (m_e + m_N)$ (m_N is the mass of the relevant nucleus), c is the speed of light in vacuum, and n is the principal quantum number in the Bohr theory. The most recent adjusted values of the fundamental constants may be found in Appendix A of this manual.

The energies of corresponding levels in hydrogen and deuterium differ by approximately 0.25% because of the different reduced masses of these two atomic systems. Precise measurements of Balmer emission spectra for these atoms permit a determination of the deuteron-to-proton mass ratio m_d/m_p to an uncertainty of 1 - 2%.

In this experiment you will assume that Eq. (1) predicts the spectrum of hydrogen *exactly* and you will use your experimental spectra to determine the mass of the deuteron through its effect on the deuterium spectrum. Do not assume the deuteron mass or even look it up in tables during the course of this experiment; it is to be determined from your experimental spectra.

The energy levels involved in the Balmer α , β , γ , and δ transitions of atomic hydrogen are shown in Fig. 1. The vacuum wavelengths $\lambda_{H \text{ vacuum}}$ of the Balmer H_α , H_β , H_γ , and H_δ lines for hydrogen, as calculated from Eq. (1), are given in Table I. To obtain these wavelengths to six significant figures, the full precision of the fundamental constants is necessary. It is essential throughout this experiment to maintain the precision of all numerical calculations to six or seven significant figures.

Eq. (1) determines transition wavelengths in vacuum, so a correction must be applied to obtain wavelengths as measured by a diffraction spectrometer in air at STP. This correction is given by

$$\lambda_{\text{air}} = \lambda_{\text{vacuum}} / n_{\text{air}}, \quad (2)$$

where n_{air} the index of refraction of air. A useful tabulation of the index of refraction of air for visible light can be found in Ref. 2. Throughout your analysis treat this index of refraction correction as exact and do not estimate its (negligible) contribution to the uncertainty in your final experimental result for m_d/m_p .

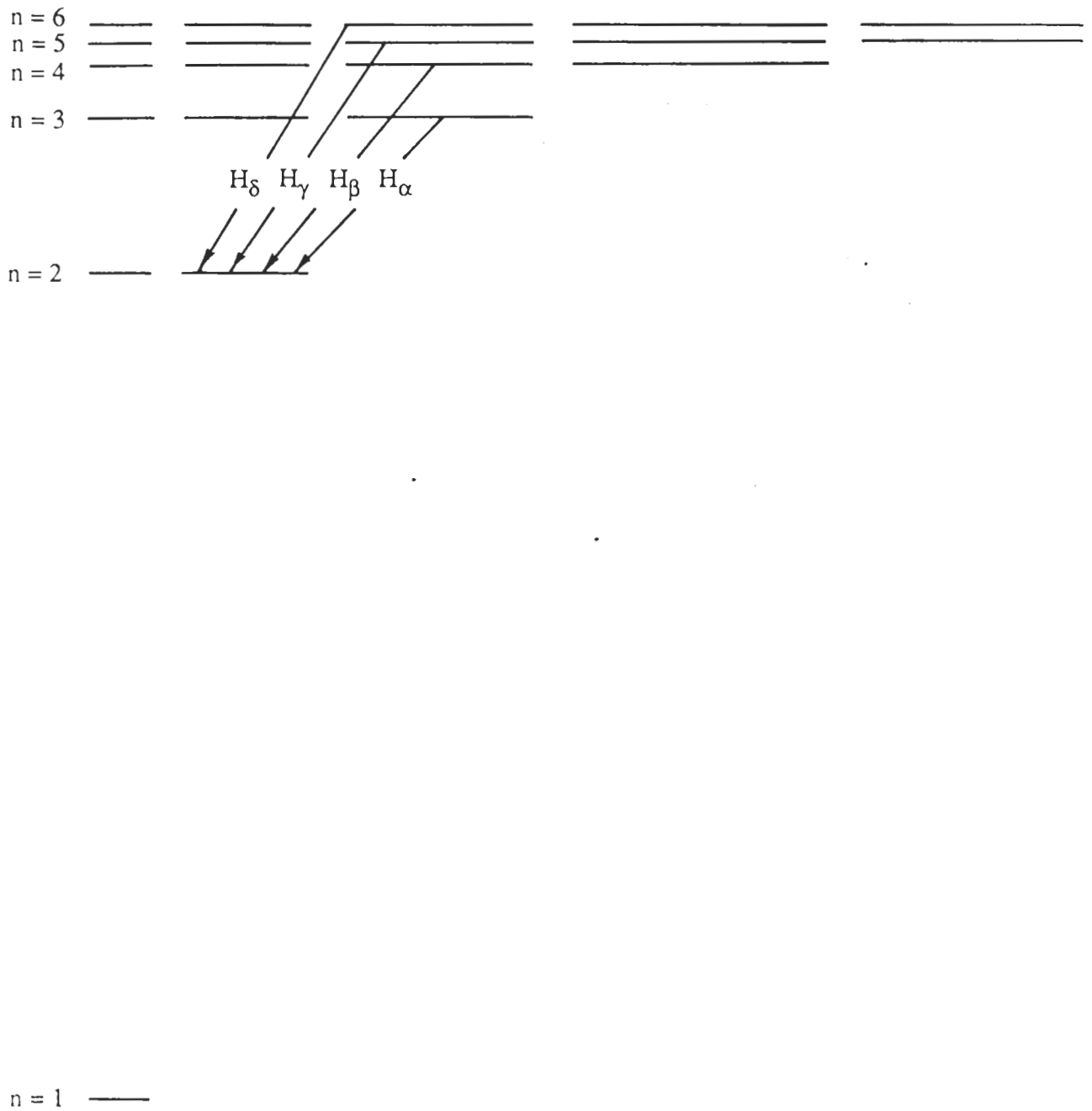


FIG. 1. Energy level diagram for atomic hydrogen in the Bohr theory, with relevant Balmer transitions indicated.

TABLE I. Wavelengths of Balmer H_α , H_β , H_γ , and H_δ lines in vacuum and in air at STP.

Transition Name	$\lambda_{H \text{ vacuum}}^a$ (Å)	$\lambda_{H \text{ air}}^b$ (Å)
H_α ($n = 3 \rightarrow n = 2$)	6564.70	6562.89
H_β ($n = 4 \rightarrow n = 2$)	4862.74	4861.38
H_γ ($n = 5 \rightarrow n = 2$)	4341.73	4340.51
H_δ ($n = 6 \rightarrow n = 2$)	4102.94	4101.78

^aCalculated from Eq. (1) of the text using the fundamental constants given in Appendix A.

^bCalculated using Eq. (2) and interpolated values of n_{air} from Ref. 2.

Procedure

Obtain high resolution spectra of the Balmer α , β , γ , and δ lines of atomic hydrogen and deuterium using the SPEX Model 1704 spectrometer and the McSPEX program³ for the IBM PC/XT. A diagram of the experimental apparatus is given in Fig. 2. Spectra for the Balmer H_α/D_α and H_δ/D_δ are given in Fig. 3(a) and 3(b). You may notice that the lineshapes of the Balmer H_α and D_α lines of Fig. 3(a) appear to be asymmetrical. The small splitting of the H_α and D_α lines is due to the fine structure splitting of the 2p state into $2p_{1/2}$ and $2p_{3/2}$ states. Fine structure effects have not been considered in Eq. (1) and will not be considered in this experiment.

Record slit and lock-in amplifier settings in your laboratory notebook. Store your four Balmer spectra on disk and plot them on 11" \times 17" graph paper using the IBM 7372 plotter.

During data acquisition be mindful of the following.

(1) It is essential to avoid high photon counting rates at the lock-in amplifier. A sure sign of excessive counting rate is "flat-topping" of the peaks in your spectra. Peaks should have a generally Gaussian or Lorentzian shape. Decreasing the slit widths or moving the H and D sources away from the beam splitter will eliminate the problem.

(2) The H and D sources produce far fewer atoms in high n states, thus the Balmer δ lines are far less intense than the Balmer α lines. It will therefore be necessary to adjust source positions, slit widths and lock-in amplifier sensitivity to obtain useful spectra for all four Balmer lines.

(3) The H and D lamps use 5000 V power supplies to which the experimenter is dangerously exposed near the metal caps at the ends of the lamps. Please turn off the H and D sources when adjusting their position so as to avoid a potentially fatal shock. Students receiving fatal shocks will not be permitted to work in the laboratory for the remainder of the term. Please take your time and be careful.

Analysis

Follow the sequence of steps outlined below.

(1) From your raw experimental spectra, record in a table entitled "Raw Experimental Data" the following quantities for each H and D peak in your four Balmer α , β , γ , and δ spectra.

(a) Peak centroid and uncertainty in its determination (you should be able to locate centroids to within ± 0.01 to ± 0.02 Å from your spectra),

(b) The full width at half maximum (FWHM) of each peak (you need not state an uncertainty in this determination),

(c) The slit setting used to obtain the appropriate spectrum.

(2) Compare your raw peak centroids for H_α , H_β , H_γ , and H_δ to the $\lambda_{H \text{ air}}$ values given in Table I. Is your agreement satisfactory, given SPEX's claim of a calibration accuracy of ± 1.0 Å

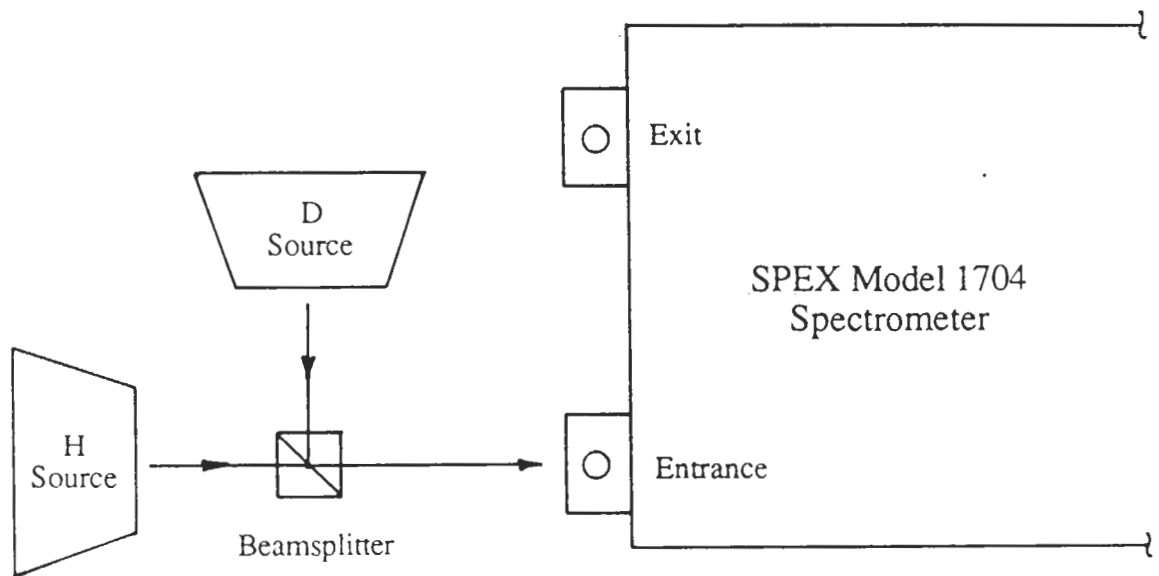


FIG. 2. Top view of the H and D sources on the optical bench. The beamsplitter is used to combine the spectral outputs of the H and D sources so that their spectra can be obtained simultaneously.

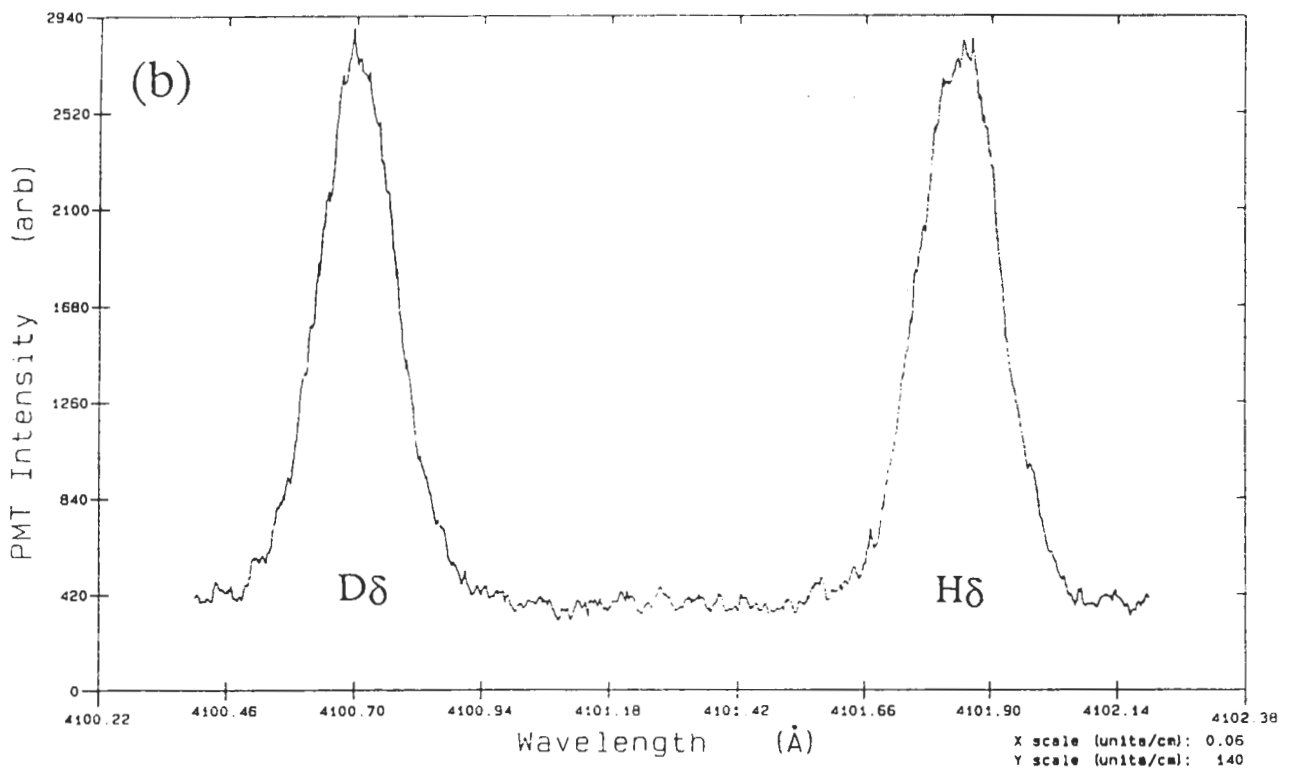
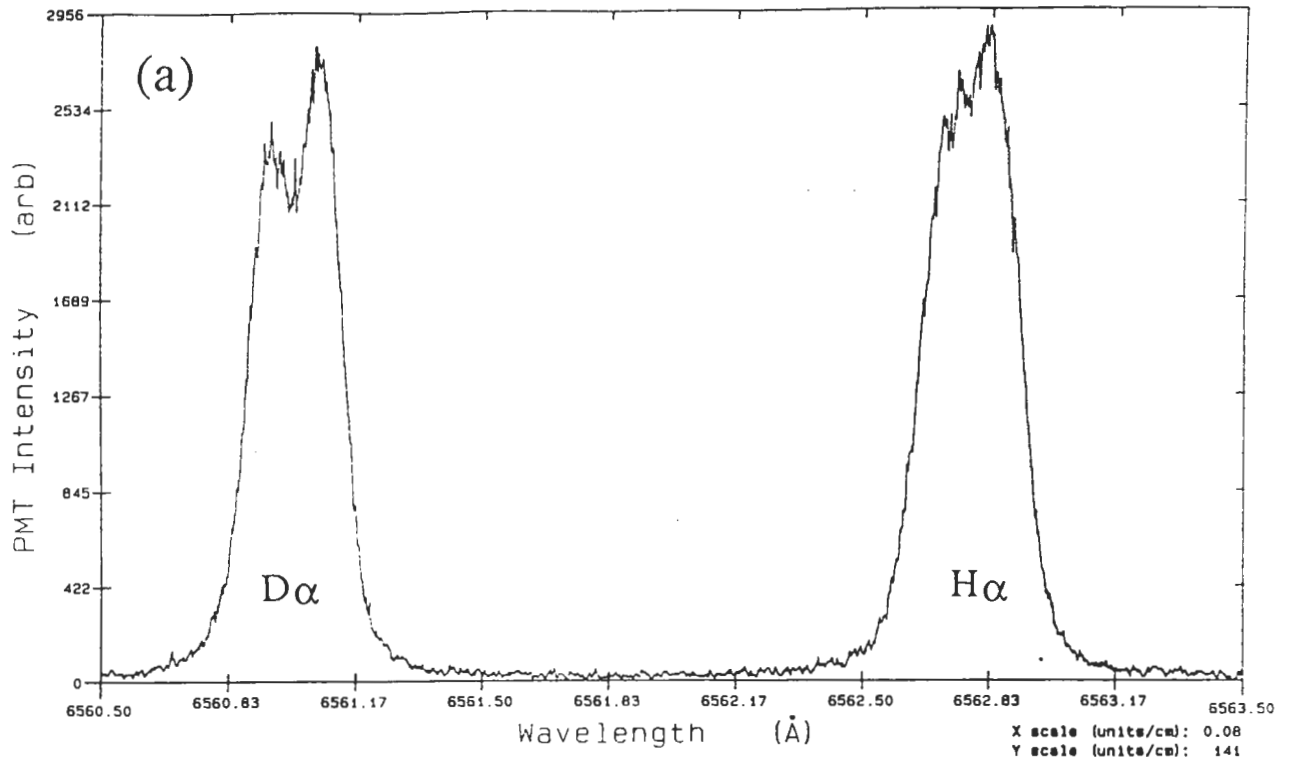


FIG. 3. Typical experimental spectra for (a) the Balmer $D\alpha$ and $H\alpha$ lines and (b) the Balmer $D\delta$ and $H\delta$ lines using the SPEX Model 1704 spectrometer.

for the SPEX Model 1704 spectrometer?

(3) Use your "Raw Experimental Data" to calculate the wavelength separation, $\Delta\lambda_{\text{air}} \equiv \lambda_{\text{H air}} - \lambda_{\text{D air}}$, between the H and D peaks of the four Balmer spectra. Also calculate the associated uncertainties $\delta(\Delta\lambda_{\text{air}})$.

(4) In your laboratory notebook show that the deuteron-to-proton mass ratio can be determined from a single Balmer transition using

$$\frac{m_{\text{d}}}{m_{\text{p}}} = \frac{A_{\text{air}}}{A_{\text{air}} - \Delta\lambda_{\text{air}}} \quad (3)$$

where $A_{\text{air}} \equiv \lambda_{\text{H air}} - \lambda_{\infty \text{ air}}$ is the *theoretical* wavelength separation (for air) between the hydrogen peak and the peak corresponding to the hypothetical, infinitely heavy nucleus case, and $\Delta\lambda_{\text{air}} \equiv \lambda_{\text{H air}} - \lambda_{\text{D air}}$ is the *experimental* H-D peak separation from part (3). Show also that the uncertainty in a $m_{\text{d}}/m_{\text{p}}$ determination is given by

$$\delta\left(\frac{m_{\text{d}}}{m_{\text{p}}}\right) = \frac{A_{\text{air}} \delta(\Delta\lambda_{\text{air}})}{(A_{\text{air}} - \Delta\lambda_{\text{air}})^2} \quad (4)$$

You will receive considerable help with the derivation of these results during the class sessions.

(5) Calculate four values of $\frac{m_{\text{d}}}{m_{\text{p}}} \pm \delta\left(\frac{m_{\text{d}}}{m_{\text{p}}}\right)$ from your four Balmer spectra. Put your results from steps (3) and (5) in a table entitled "Final Deuteron-to-Proton Mass Ratio Results". Compute a weighted mean of your $m_{\text{d}}/m_{\text{p}}$ results and its associated uncertainty. Compare your final value for $m_{\text{d}}/m_{\text{p}}$ with the currently accepted value.

Additional Comments:

Begin working on wavelength calculations from Eq. (1) immediately. Be sure you can calculate the numbers of Table I right down to the last significant figure. You should agree with every number in those values if you maintain the required number of significant figures.

Use a programmable calculator or computer to make your analysis less time-consuming. If you use a computer, you must be absolutely certain that it can perform the necessary calculations with the required precision.

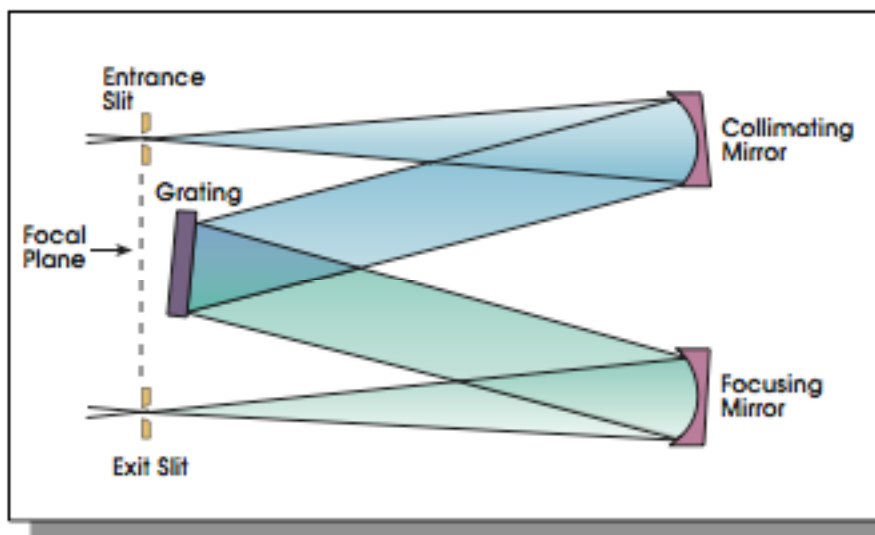
An important goal of this experiment is to acquaint you with the theory of uncertainty propagation in experimental physics. Give careful thought to your uncertainty estimates and the procedures used to determine the uncertainty in your final value for the deuteron-to-proton mass ratio.

References

1. R. Eisberg and R. Resnick, *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles* (John Wiley & Sons, New York, 1985), pp. 100-107.
2. *Handbook of Chemistry and Physics*, edited by R.C. Weast, 63rd ed. (CRC Press, Boca Raton, FL, 1983), p. E-379.
3. E.B. Anthony, *The McSPEX Project: Laser Spectroscopy Laboratory Interface Program*, B.A. thesis, Middlebury College, 1987.

APPENDIX: Description of the Reed Spectrometer

The SPEX Model 1000M spectrometer in our laboratory is a research-grade instrument capable of providing high-resolution spectra in the range from 4000 to 8000 Angstrom (A).



The figure above shows the location of the various optical components inside the spectrometer housing. The positions of the grating and two mirrors in this spectrometer are characteristic of what is called a Czerny-Turner configuration. In this configuration, the incident light (which is to be analyzed) enters through a narrow entrance slit that lies at the focus of a concave mirror of 1.0 m focal length. The concave mirror reflects the light to produce a cylindrical beam of parallel rays that illuminate the full surface of a rotatable diffraction grating. The grating diffracts the light, dispersing the various wavelengths within the original incident light to different angles in the horizontal plane. The “rainbow” of light is collected by a second concave mirror that focuses the light rainbow on the wall that contains the exit slit. By rotating the grating, one can control which particular wavelength (from the rainbow) passes through the exit slit. Just behind this slit is the entrance to a photomultiplier tube (PMT). The PMT produces an electronic pulse each time a photon is incident upon its front surface. These pulses are counted by the Stanford Research System SR400 Photon Counter for a predetermined amount of time, and this photon count at the given wavelength is proportional to the light intensity at that wavelength. A spectrum (i.e., light intensity vs. wavelength) is obtained by programming the spectrometer (using LabVIEW software) to rotate the grating through a sequence of angles so that the photon count (for a predetermined time interval) can be determined at each of a sequence of light-wavelengths.

The table on the next page summarizes the manufacturer's specifications for the Model 1000M spectrometer. The wavelength resolution of the spectrometer is defined as the relative wavelength separation between two infinitely narrow peaks, such that two peaks, rather than just one, can be readily discerned in an experimental spectrum. The resolution depends on how wide the variable-sized slits are set by the user and the spectrometer's dispersion specification (= nm of wavelength resolution per mm of slit opening). From the table we see that the 1000M's dispersion is 0.8 nm/mm. Thus, if the user sets the

entrance and exit slits to their minimum width of $10 \mu\text{m} = 0.01 \text{ mm}$, then the ultimate resolution of the instrument will be obtained, which is $(0.8 \text{ nm/mm})(0.01 \text{ mm}) = 0.008 \text{ nm}$, or equivalently, 0.08 \AA ($1 \text{ \AA} = 0.1 \text{ nm}$). Another important specification of a spectrometer is the absolute accuracy of its wavelength readout mechanism. After learning that the relative wavelength resolution of the Model 1000M can be as low as 0.08 \AA , it may be surprising to find in the table that the absolute accuracy in wavelength measurement is only $\pm 0.5 \text{ \AA}$. Fortunately, in many experiments an absolute wavelength accuracy is sufficient and in other experiments, which require higher precision, the analysis can be designed to use (relative) wavelength differences for making comparisons to theory.

M Series Spectrometers

4.0 Specifications

	500M	750M	1000M	1250M
Focal Length, meters	0.5	0.75	1.0	1.25
Aperture	f/4 ¹ f/6.9 ²	f/6 ¹ f/10.4	f/8 ¹ f/13.9 ²	f/9 ³ f/11 ¹
Spectral Range, \AA (mechanical atmosphere limited)	0-15000	0-15000	0-15000	0-15000
Dispersion, nm/mm	1.6	1.1	0.8	0.65
Resolution, nm	0.02	0.01	0.008	0.006
Multichannel Coverage, nm over 25 mm	40	27.5	20	16.25
Accuracy, nm	± 0.05	± 0.05	± 0.05	± 0.05
Repeatability, nm	± 0.005	± 0.005	± 0.005	± 0.005
Drive Step Size, nm	0.00025	0.00025	0.00025	0.00025
Dimensions Inches (cm)				
width	12 (30.5)	12.25 (31.1)	13.75 (35)	17.75 (45)
length	23 (58.4)	33 (83.8)	43 (109.2)	53 (134.6)
height	13 (33.0)	13 (33.0)	14.5 (36.8)	14.5 (36.8)
Weight lbs. (kg)	82 (37.3)	105 (47.8)	160 (72.7)	210 (96.9)

1. with 110 x 100 mm grating
2. with 64 x 64 mm grating
3. with 120 x 140 mm grating