

An undergraduate laboratory experiment on quantized conductance in nanocontacts

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We describe an undergraduate laboratory experiment on conductance steps observed to occur near integer multiples of $2e^2/h$ as nanocontacts form and break between gold wires in loose contact. This experimental arrangement was first described by Costa-Krämer *et al.* [Surf. Sci. **342**, L1144 (1995)]. A simple op-amp circuit in conjunction with an inexpensive storage oscilloscope suffice to observe the steps. The experiment may be extended by interfacing to a computer, which accumulates a histogram of conductance values as the wires are brought into and out of contact many times. The histogram shows peaks near integer multiples of $2e^2/h$. We emphasize the pedagogical issues involved in bringing an experiment from forefront condensed-matter physics research into the undergraduate laboratory. © 1999 American Association of Physics Teachers.

I. INTRODUCTION

An increasingly large and important field of physics is concerned with the fabrication and properties of *nanostuctures*—structures as small as a few to hundreds of atoms across.^{1–3} Such structures display a host of electrical, mechanical, and optical properties not seen in larger devices, properties which typically manifest the underlying quantum mechanics in striking ways. Nanostructures will play a role in the next generation of microelectronics, and some of their emergent properties may form the bases for future technologies.

Unfortunately, nanostructure physics has been largely inaccessible to the typical undergraduate physics laboratory. Nanostructure fabrication has typically involved electron-beam lithography, either to directly create metal structures⁴ or to define gate structures atop molecular beam epitaxy (MBE)-grown semiconductor heterojunctions.⁵ Another route uses scanning probe microscopy to create and study structures like point contacts.⁶ Additionally, most of these experiments had to be carried out at cryogenic temperatures, often well below 1 K.

Recently, it was shown that a simple, room-temperature apparatus can be used to explore a key nanostructure effect:⁷ the quantization of electrical conductance in multiples of $2e^2/h$. The apparatus consists of two wires that are gently vibrated to bring them into and out of contact with each other. A constant voltage is applied to the wires and the resulting current is monitored using a storage oscilloscope. Transient nanosize contacts are formed and destroyed between the wires on a millisecond time scale, resulting in a staircase pattern on the oscilloscope as the conductance decreases through integer multiples of $2e^2/h$.

Newly available, inexpensive digital oscilloscopes, coupled with graphical general-purpose laboratory software have made this project particularly accessible for the teaching laboratory. In this paper we describe in detail our implementation of a quantized-conduction experiment in a laboratory course for third- and fourth-year undergraduate physics majors.

II. LABORATORY SETTING

This experiment was constructed as part of a lab course required of all physics majors, which attempts to capture

some of the flavor of basic research in an upper-level undergraduate course. The students work in collaborative teams of two or three; each team must do some library research and make a brief oral presentation before any laboratory work. Over the course of a semester a student participates as a team member in six projects chosen from about a dozen available experiments. As an open-ended project for which the underlying physics is still under debate in the research community, the quantized-conductance experiment is appropriate for the more motivated students to tackle near the end of the semester.

In an effort to more closely duplicate the research environment, this course was recently updated with desktop computers running the LABVIEW graphical data acquisition software package.⁸ Each computer was outfitted with a data acquisition board as well as a general purpose interface bus (GPIB) interface. The LABVIEW software is able to control and read the internal board and external GPIB instruments, perform some data analysis and presentation, and store the data for later processing. Computerized data acquisition is used for only about half of the projects in the course—those for which it offers some particular benefit. The more advanced part of the quantized-conductance experiment falls in this category as it involves the acquisition and analysis of a large quantity of data.

The graphical programs written for LABVIEW are easily used by the average student, who might also learn how to make some slight modifications. Creating new LABVIEW programs requires a more detailed understanding of the software, which we do not require of the students in this class. Students with similar previous programming experience, or who are willing to devote extra time to the course, are encouraged to learn how to use the graphical programming environment. The LABVIEW program used for the quantized conductance experiments described here was written by one of the authors (E.F.) while a student in the course.⁹ It should be emphasized that the LABVIEW system is by no means necessary to carry out this quantized-conductance project. The required equipment is some type of storage oscilloscope, or else a reasonably fast data acquisition card in a computer.

III. THE PHYSICS OF QUANTIZED CONDUCTANCE

Discussion of conductance quantization in nanostructures has only recently begun to appear in undergraduate physics

texts.¹⁰ The phenomenon was discovered serendipitously in experiments on point contacts formed in a two-dimensional electron gas in a semiconductor heterojunction.^{5,11} A rigorous explanation requires concepts from solid-state physics (Fermi surface, density of states) that are not always taught at the undergraduate level.¹² However, the following rough argument has been offered:¹ A nanosized contact acts like a waveguide for electrons. Just as for a microwave waveguide, there will in general only be a finite, integer number of occupied modes “above cutoff,” that are able to propagate in the waveguide. As the size of the contact is varied, the number of allowed modes jumps discontinuously from one integer to the next. For each allowed mode, the nanocontact acts essentially like a one-dimensional wire with conductance $2e^2/h$. The standard arguments for this value of the conductance are framed in terms of a cancellation between the electron group velocity and the density of states at the Fermi level. We have reformulated this argument in terms that we feel should be accessible to most undergraduate physics students, as follows.

The current in a wire is the velocity of the electrons multiplied by the charge and number per unit length, $I = veN/L$. The conductance is the current divided by the voltage, $G = I/V = veN/LV$. The drop in potential energy for an electron going from one end of the wire to the other is $\Delta E = eV$, hence $G = ve^2N/L\Delta E$. Thus the problem reduces to computing the number N of electrons in a one-dimensional wire of length L that contribute to conduction when the potential energy difference from end to end is ΔE .

At this point the Pauli exclusion principle and elementary quantum mechanics for a particle in a box must be invoked. The quantum states of the terminals connected to the two ends of the wire are filled up to some energy (the Fermi energy), with two electrons per state. The Fermi energy in one terminal is higher than in the other terminal by ΔE , and this causes current to flow. Electrons in a range ΔE below the Fermi energy in one terminal can flow into unoccupied states in the other terminal. Thus the number N of electrons that contribute to conduction is twice the number of quantum states in this range of energies ΔE .

In a box of length L , the de Broglie wavelength of an electron can only take on discrete values $\lambda_n = L/n$, $n = 1, 2, 3, \dots$. The velocity is $v = h/\lambda m$, so the velocity can only have discrete values $v_n = nh/Lm$. From this one computes the number of electrons in a velocity range Δv to be $N = 2Lm\Delta v/h$. The kinetic energy of an electron of mass m is $E = mv^2/2$, so $\Delta E = mv\Delta v$, allowing the number of electrons contributing to conduction to be expressed as $N = 2L\Delta E/vh$. This immediately gives $G = 2e^2/h$.

Students with some exposure to solid state physics may know that the effective mass m of an electron in a metal or semiconductor is often very different from the bare electron mass m_e . As m cancels out in the argument above, the conductance steps are not shifted from integer multiples of $2e^2/h$ by these effective-mass effects, no matter how strong they are.

In the original experiments on semiconductors,⁵ the electron wavelength at the Fermi energy λ_F was much larger than the atomic spacing and there could be little doubt that the nanostructure was acting as an electronic waveguide. In metals, as used for the experiments described here, λ_F is comparable to the atomic spacing. A debate continues in the

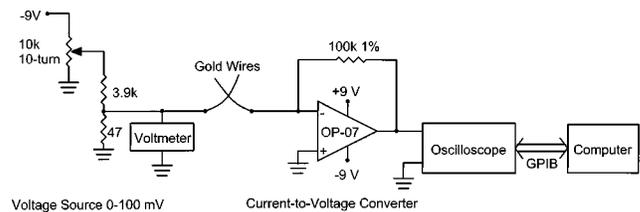


Fig. 1. Experimental setup. The voltage source and current-to-voltage converter may be built together in a small metal box, along with two 9-V batteries and a power switch. We selected the type OP-07 op-amp for its low noise and high input impedance, but many other types could be substituted. For greater flexibility, the feedback resistor may be made switch selectable in the range 1 k Ω –1 M Ω . For the value shown here (100 k Ω) the gain of the current-to-voltage converter is -10^5 V/A.

research literature as to whether the observed conductance steps truly reflect the physics of conductance quantization as outlined above, or rather the discrete nature of few-atom contacts and their rearrangements as the contact is narrowed.^{6,13–17} In Ref. 13 it is concluded that both conductance quantization and atomic rearrangements contribute to the pattern of conductance steps. Atomic force microscopy experiments and molecular-dynamics simulations show oscillations in the *force* between the wires, associated with atomic rearrangements, that appear to be correlated with the conductance steps.^{6,14} Two groups have questioned the assumption that the observed peaks in conductance histograms directly reflect the number of allowed conduction modes.^{15,16} Conversely, an experiment on nanocontacts formed of bismuth (a semimetal with large λ_F) has been interpreted as evidence that the conductance steps and histogram peaks truly reflect quantization of electronic conduction modes.¹⁷ Disagreements like this have pedagogical value in the context of an undergraduate lab course—most of our students have had little exposure to the sort of debate that is common at research frontiers.

IV. EXPERIMENTAL SETUP

The experimental setup is modeled closely on that described in Ref. 7. A constant voltage source in the range of millivolts is connected to one of a pair of fine gold wires in loose contact with one another. The other wire is connected to the virtual-ground input of an op-amp current-to-voltage (I – V) converter, and the output of the converter is fed to a storage oscilloscope (Fig. 1). It is the recent advent of inexpensive digital storage oscilloscopes¹⁸ that makes this project feasible for the average undergraduate physics lab.

To avoid excessively shifting the conductance steps, the output resistance of the voltage source should be no more than about 50 Ω .¹⁹ Figure 1 shows a simple voltage source with 47- Ω output resistance that can be constructed in the same box as the I – V converter. For the experiments described here we used the dc-offset output of an HP 33120A function generator as a voltage source. We used voltages of 10–20 mV; much larger voltages would potentially lead to substantial electron-heating effects whereas much smaller voltages would require more careful shielding of the entire circuit.

The wires in our setup were gold, 0.05 mm in diameter, and several centimeters long. They were soldered to short lengths of larger bare copper wire so contact could be made using ordinary electronic clip leads. In the original experi-

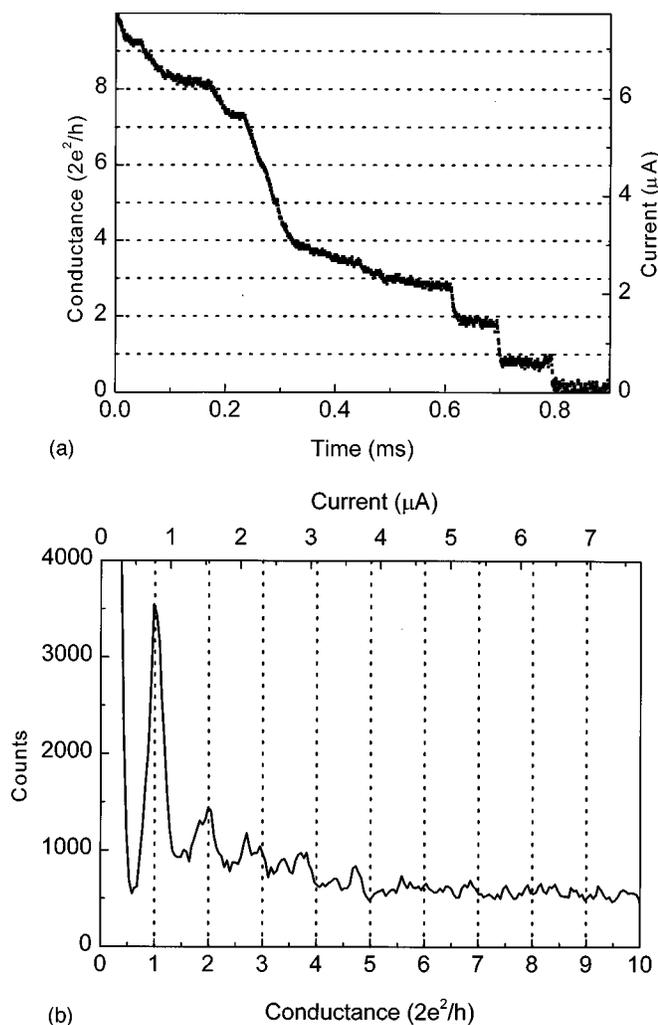


Fig. 2. Data taken with 10 mV across the wires. (a) A representative trace showing current (right axis) and conductance (left axis) vs time as contact breakage occurs. (b) Histogram of current values accumulated from 104 traces.

ment conductance steps were seen in nanocontacts between much larger wires, both of gold and other metals.⁷ Recently, room-temperature conductance steps have been observed in a variety of other geometries, such as a tip suspended over a plate¹⁴ or the contacts of a commercial electromechanical relay.²⁰ The circuitry and data acquisition setups described here should be equally useful for these alternative arrangements. Although the diameter of the wires (or other contacts) used is not critical, it appears that gold consistently gives the largest number of clear conductance steps.⁷ This may reflect the malleability and/or the freedom from surface oxidation of gold. For an undergraduate lab it is probably best to start with gold wires and to progress to other materials for a more in-depth experiment.

As much of the circuit as possible must be shielded to avoid electromagnetic interference. We built the I - V converter in an aluminum box with BNC connectors and used coaxial cable for most of the other wiring apart from the leads near the gold wires. We found it necessary to turn off the fluorescent room lights as their electronic ballasts generated considerable interference.

The gold wires must be supported in such a way that a slight vibration, such as tapping on the table, will bring them

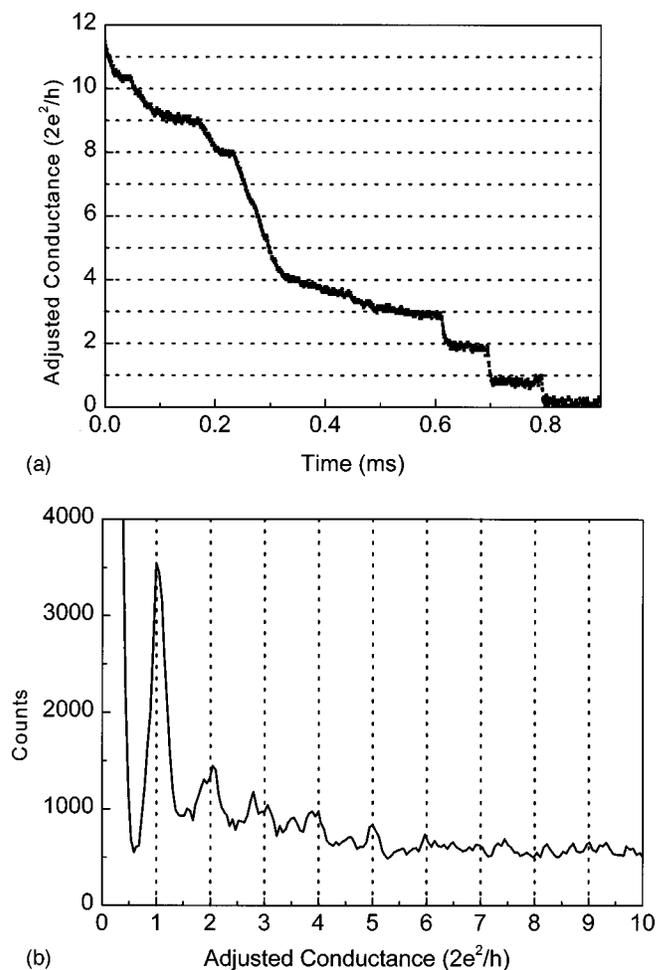
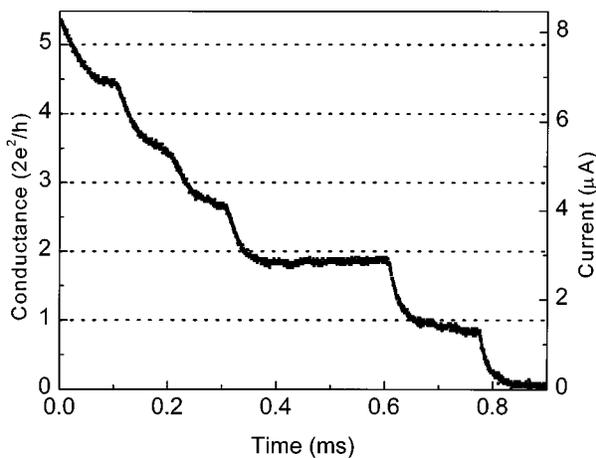


Fig. 3. The same data as shown in Fig. 2, but corrected for the voltage source output resistance and an additional residual resistance $R_{\text{res}} = 100 \Omega$. This value of R_{res} was chosen to move the histogram peaks closer to the expected quantized values.

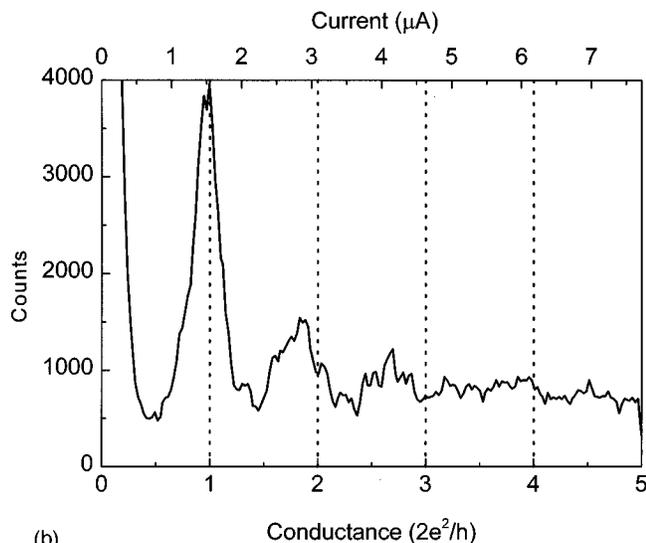
into and out of contact. There are many ways of achieving this and we find that the students are able to devise suitable arrangements on their own. For the experiments reported here the leads to the gold wires were taped to a piece of paper that was set upon some compliant cardboard packing material.

As a pre-lab exercise the students are required to select a suitable vertical scale setting for the oscilloscope so that a known number of conductance steps will fill the vertical range. This forces them to deal with the somewhat unfamiliar concept of conductance, and to consider the functioning of the I - V converter. The optimal time base setting for the oscilloscope depends upon the wires used and their method of suspension. Settings of $10 \mu\text{s}$ - $1 \text{ ms}/\text{division}$ were suitable for our experiments with 0.05-mm -diam wires. The oscilloscope must be set for normal-mode dc triggering, with the threshold and slope chosen to trigger on connecting or disconnecting events as desired.

The first part of this project, the observation and measurement of conductance steps, can be accomplished without a computer. We used a GPIB interface to a desktop computer running LABVIEW to record the conductance traces as disk files for later analysis and display. This also permitted the accumulation of histograms of the conductance values as a more advanced part of the project. The LABVIEW program



(a)



(b)

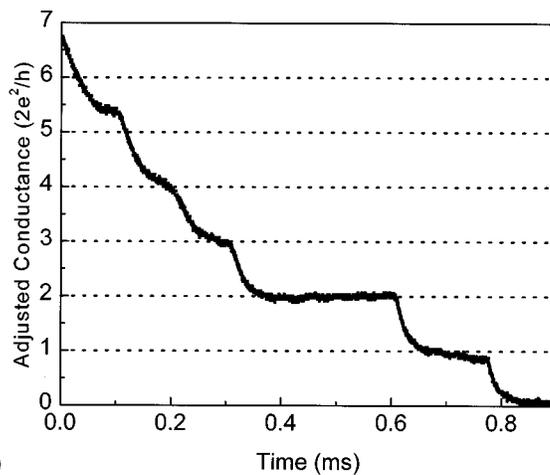
Fig. 4. (a) Single time trace and (b) histogram of current values accumulated from 103 traces for 20-mV excitation voltage.

developed for this experiment controls the oscilloscope's data acquisition, allowing the user to set parameters such as input channel, trigger mode and level, and vertical and horizontal scale and position, on the computer screen. When the oscilloscope triggers, the voltage trace is read in and displayed on the computer screen. A dialog box appears allowing the user to optionally add the trace to the histogram. Normally some selection criterion is applied, for example, accepting all traces that fall to zero within a specified time interval. The more advanced students can develop their own selection criteria and explore the disadvantages of using too strict or lax ones.

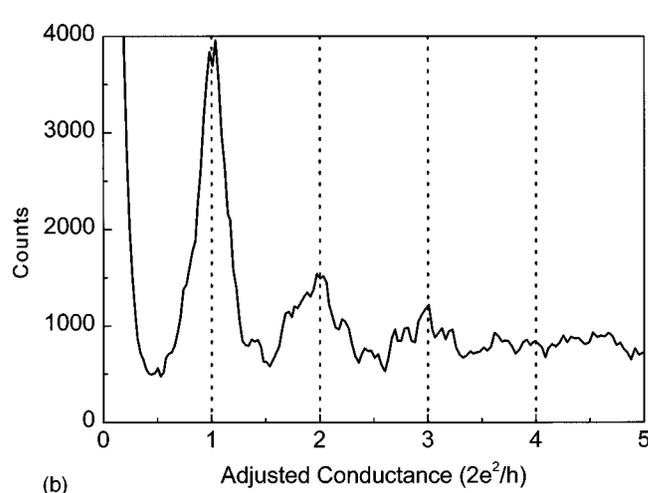
The histogram is created by defining a set of bins, each covering an equal-sized voltage interval. For each time trace that is selected for addition into the histogram, the number of time points in the trace that fall within each bin are added to the count for that bin. The creation of the histogram is a built-in function in LABVIEW, but the bin size must be carefully selected. Unless it is an integral multiple of the minimum voltage step imposed by the oscilloscope, artifacts will appear in the histogram in the form of regularly spaced spikes or dips.

V. RESULTS AND DISCUSSION

A sample current decay trace with 10 mV across the wires is shown in Fig. 2. Also shown is a histogram accumulated



(a)



(b)

Fig. 5. Data of Fig. 4 corrected for voltage source output resistance and an additional residual resistance $R_{\text{res}} = 450 \Omega$.

from 104 traces for this input voltage. There is clearly a peak at G_0 , and smaller peaks are found near multiples of G_0 up to 5. The third, fourth, and fifth peaks, however, appear to be shifted to slightly lower conductance values. This is consistent with previous research, which showed that the peaks are shifted as if there were a residual resistance in series with the nanocontacts. This resistance was originally presumed to be due to the lead geometry, but it has more recently been conjectured that the residual resistance is due to internal disorder in the nanocontacts.¹⁵ The measured conductance G and residual resistance R_{res} are used to compute a corrected conductance G_c via

$$G_c = (G^{-1} - (R_{\text{res}} + R_{\text{out}}))^{-1}.$$

Here R_{out} is the output resistance of the voltage source, 50 Ω for our setup. As R_{res} is not known *a priori*, it is varied experimentally to find the value that best brings the histogram peaks into registry with the quantized conductance values. As shown in Fig. 3(b), our histogram data with 10-mV excitation voltage are best fit by $R_{\text{res}} = 100 \Omega$. Figure 3(a) shows the effect of correcting for the same value of R_{res} on a single trace with this excitation voltage.

This project inevitably confronts students with the problem of distinguishing true conductance steps and histogram peaks from meaningless fluctuations. It also offers an example of a classic solution to this type of problem: Vary one

of the input parameters, and check if the supposed effect varies as it should. Comparing Figs. 2 and 4, it is clear that doubling the excitation voltage to 20 mV results in a doubled spacing between the histogram peaks. The residual resistance that best fits the data with 20-mV excitation is $R_{\text{res}} = 450 \Omega$ (Fig. 5). To our knowledge, a dependence of R_{res} on excitation voltage has not been reported previously. It may reflect electron heating effects,¹² as eV/k_B is close to room temperature at the larger excitation voltage used.

In conclusion, we have found it relatively easy and inexpensive to construct an undergraduate laboratory project on conductance quantization in transient nanocontacts. The project offers students valuable exposure to a current research topic in condensed-matter physics, which ordinarily is accorded little space in the undergraduate curriculum. This project also carries useful lessons on triggering and data selection, distinguishing physical effects from noise, and the use of computers for statistical data acquisition.

ACKNOWLEDGMENT

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²For a basic overview see D. Goldhaber-Gordon, M. S. Montemerlo, J. C. Love, G. J. Opiteck, and J. C. Ellenbogen, "Overview of Nanoelectronic Devices," *Proc. IEEE* **85** (4), 521–540 (1997).

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⁸LABVIEW is a trademark of National Instruments, Austin, TX.

⁹The LABVIEW program developed for these experiments may be obtained by contacting the second author at candela@phast.umass.edu.

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¹¹H. van Houten and C. Beenakker, "Quantum Point Contacts," *Phys. Today* **49** (7), 22–27 (1996).

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¹⁵P. García-Mochales and P. A. Serena, "Disorder as Origin of Residual Resistance in Nanowires," *Phys. Rev. Lett.* **79** (12), 2316–2319 (1997).

¹⁶A. I. Yanson and J. M. van Ruitenbeek, "Do Histograms Constitute a Proof for Conductance Quantization?" *Phys. Rev. Lett.* **79** (11), 2157–2157 (1997).

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¹⁸We used a Tektronix model TDS210 oscilloscope with model TDS2CM communications extension module.

¹⁹The output resistance of the source is effectively in series with the nanocontact. On the tenth conductance step, the nanocontact resistance is $1/(10 \times 2e^2/h) \approx 1.3 \text{ k}\Omega$, so the shift due to a 50Ω output resistance is 4%. The shift is correspondingly less for the lower conductance steps.

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CALCULUS OF VARIATIONS

When I was professor at one of our greater universities, a student came to me and said: "Professor, are we supposed to memorize all of your lectures?" I fear this question drove me almost to the realm of profanity, for I replied: "My God, man, such a thing is impossible. Whenever did you get that idea?" "Well," he said, "I fear that unless I do memorize them I shall be unable to pass the examination, because you are using the calculus of variations and I have never had a course in the calculus of variations." "Fine," said I, "neither have I had a course in the calculus of variations. Moreover, I have never had a course on the subject (electrodynamics) I am lecturing to you about now." I fear the poor fellow was completely flabbergasted, but in time he saw the light and became a very faithful disciple.

W. F. G. Swann, "The Teaching of Physics," *Am. J. Phys.* **19**(3), 182–187 (1951).