Problem 33.1 (Griffiths 5.7)

a. $\psi(x_1, x_2, x_3) = \boxed{\psi_a(x_1)\psi_b(x_2)\psi_c(x_3)}$ for distinguishable particles.

b. We have to symmetrize in all three states:

$$\psi(x_1,x_2,x_3) = A\big[\psi_a(x_1)\psi_b(x_2)\psi_c(x_3) + \psi_a(x_2)\psi_b(x_1)\psi_c(x_3) + \psi_a(x_3)\psi_b(x_2)\psi_c(x_1)$$
$$+ \psi_a(x_1)\psi_b(x_3)\psi_c(x_2) + \psi_a(x_2)\psi_b(x_3)\psi_c(x_1) + \psi_a(x_3)\psi_b(x_1)\psi_c(x_2)\big]$$

w/ $A = 1/\sqrt{6}$.

c. Form:

$$\begin{vmatrix} \psi_a(x_1) & \psi_b(x_1) & \psi_c(x_1) \\ \psi_a(x_2) & \psi_b(x_2) & \psi_c(x_2) \\ \psi_a(x_3) & \psi_b(x_3) & \psi_c(x_3) \end{vmatrix} = \psi_a(x_1)\big(\psi_b(x_2)\psi_c(x_3) - \psi_b(x_3)\psi_c(x_2)\big) - \psi_b(x_1)\big(\psi_a(x_2)\psi_c(x_3) - \psi_c(x_2)\psi_a(x_3)\big) + \psi_c(x_1)\big(\psi_a(x_2)\psi_b(x_3) - \psi_a(x_3)\psi_b(x_2)\big).$$

so

$$\psi(x_1 x_2 x_3) = A\big[\psi_a(x_1)\psi_b(x_2)\psi_c(x_3) - \psi_a(x_1)\psi_b(x_3)\psi_c(x_2) - \psi_a(x_2)\psi_b(x_1)\psi_c(x_3)$$
$$+ \psi_a(x_3)\psi_b(x_1)\psi_c(x_2) + \psi_a(x_2)\psi_b(x_3)\psi_c(x_1) - \psi_a(x_3)\psi_b(x_2)\psi_c(x_1)\big].$$

w/ $A = 1/\sqrt{6}$ again.

Problem 33.2 (Griffiths 5.16)

a. $d = 8.96\ \mathrm{g/cm^3}$ & $63.5\ \mathrm{gm/mole} = M$

$E_F = \frac{\hbar^2}{2m}(3\rho\pi^2)^{2/3}$ $\qquad \rho = \frac{N}{V}$ is in $\frac{\text{atoms}}{\text{cm}^3} = \frac{\text{atoms}}{\text{mole}} \cdot \frac{\text{mole}}{\text{gm}} \cdot \frac{\text{gm}}{\text{cm}^3}$

$= N_A \cdot \frac{d}{M} \approx 6.02\times10^{23} \cdot \frac{8.96\ \mathrm{g/cm^3}}{63.5\ \mathrm{g/mol}}$

$= 8.5\times10^{22}\ \frac{\text{atoms}}{\text{cm}^3} \cdot \frac{(100\ \text{cm})^3}{\text{m}^3} = 8.5\times10^{28}\ \frac{\text{atoms}}{\text{m}^3}.$

& $E_F = \frac{(1.05\times10^{-34}\ \mathrm{Js})^2}{2(\underbrace{9.11\times10^{-31}\ \mathrm{kg}}_{e^-\ \text{mass}})}\left(3 \cdot 8.5\times10^{28}\ \frac{\text{atoms}}{\text{m}^3} \cdot \pi^2\right)^{2/3} \approx 1.12\times10^{-18}\ \mathrm{J}$

so $E_F = 1.12\times10^{-18}\ \mathrm{J} \cdot \frac{6.24\times10^{18}\ \mathrm{eV}}{\mathrm{J}} = \boxed{6.99\ \mathrm{eV}}$

b. Take $E_F = 1.12\times10^{-18}\ \mathrm{J} = \frac{1}{2}(9.11\times10^{-31}\ \mathrm{kg})\,v^2 \Rightarrow \boxed{v \approx 1.57\times10^6\ \mathrm{m/s}}$ (so $v/c = .005$, non-relativistic).

c. $k_B = 1.38\times10^{-23}\ \mathrm{J/K}$, & $k_B T = E_F \Rightarrow 1.38\times10^{-23}\ \mathrm{J/K} \cdot T = 1.12\times10^{-18}\ \mathrm{J}$
$\Rightarrow \boxed{T = 81000\ \mathrm{K}}$

d. $P = \frac{(3\pi^2)^{2/3}\hbar^2}{5m}\rho^{5/3} \approx \boxed{3.8\times10^{10}\ \mathrm{N/m^2}.}$

m: e^- mass; ρ: from part a.

Problem 33.3 Griffiths 5.34

In a two-dimensional square well, we have: $\rho_k = \frac{l_x l_y}{\pi^2} = \frac{A}{\pi^2}$
and the total # of e^- is: Nq, so

$$\underbrace{\frac{1}{4}\pi k_F^2}_{\text{one quadrant}} \cdot \rho_k = \frac{Nq}{2} \Rightarrow k_F^2 = 2\frac{Nq}{\pi \rho_k} = 2\frac{Nq}{A} \cdot \pi = 2\sigma\pi$$

then $k_F = \sqrt{2\sigma\pi}$

The Fermi energy is the energy associated w/ this circle:

$$\boxed{E_F = \frac{\hbar^2}{2m} k_F^2 = \frac{\hbar^2}{m}\sigma\pi}$$