Problem 3.1

a. $\left|\frac{\partial f}{\partial t}\right| = \boxed{\frac{F}{T}}$ & $\left|\frac{\partial^2 f}{\partial x^2}\right| = \boxed{\frac{F}{L^2}}$

b. eqn.: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}$

units: $\frac{|\hbar|^2}{M}\cdot\frac{1}{L^2} + E = \frac{|\hbar|}{T}$ $\Rightarrow$ $|\hbar|^2 + \underbrace{ML^2\cdot E}_{} = \frac{ML^2}{T}\cdot|\hbar|$

$= ML^2\cdot(ML^2/T^2) = \left(\frac{ML^2}{T}\right)^2$

So we have $|\hbar^2| + \left(\frac{ML^2}{T}\right)^2 = \frac{ML^2}{T}\cdot|\hbar|$

& we must have $\boxed{|\hbar| = ML^2/T \doteq E\cdot T}$ to get all terms to have the same dimension.

Problem 3.2

a. Start w/ $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi = i\hbar\frac{\partial\psi}{\partial t}$ & let $\psi(x,t) = \psi(x)\phi(t)$, then:

$$-\frac{\hbar^2}{2m}\psi''\phi + V\psi\phi = i\hbar\psi\dot{\phi}$$

divide both sides by $\psi(x)\phi(t)$ to get

$$\underbrace{-\frac{\hbar^2}{2m}\frac{\psi''(x)}{\psi(x)} + V(x)}_{\text{func. of } x \text{ only}} = \underbrace{i\hbar\frac{\dot{\phi}(t)}{\phi(t)}}_{\text{func. of } t \text{ only}}$$

and we must have each side seperately equal to E, a constant.

$$-\frac{\hbar^2}{2m}\frac{\psi''(x)}{\psi(x)} + V(x) = E = i\hbar\frac{\dot{\phi}(t)}{\phi(t)}$$

$$|E| = |\hbar|\cdot\frac{1}{T} = \frac{ML^2}{T^2} = \text{energy}$$

b. We can solve $i\hbar\frac{\dot{\phi}(t)}{\phi(t)} = E \Rightarrow \dot{\phi}(t) = \frac{E}{i\hbar}\phi(t) = -i\frac{E}{\hbar}\phi(t)$

so

$$\boxed{\phi(t) = \phi_0 e^{-iEt/\hbar}}$$

& $-\frac{\hbar^2}{2m}\frac{\psi''(x)}{\psi(x)} + V(x) = E \Rightarrow \boxed{-\frac{\hbar^2}{2m}\psi''(x) + V(x)\psi(x) = E\psi(x)}$

Problem 3.3

a. For $\vec{p} = \alpha \vec{v}_1 + \beta \vec{v}_2$, we have:

$$\mathbb{A}\vec{p} = \alpha \mathbb{A}\vec{v}_1 + \beta \mathbb{A}\vec{v}_2 = \alpha\lambda\vec{v}_1 + \beta\lambda\vec{v}_2 = \lambda(\alpha\vec{v}_1 + \beta\vec{v}_2) = \lambda\vec{p}$$

so

$\mathbb{A}\vec{p} = \lambda\vec{p}$ & $\vec{p}$ is an eigenvector w/ eigenvalue λ. ✓

b. We want $\vec{v}_1 \cdot \vec{p} = \varnothing$, and we can pick α & β to enforce this constraint:

$$\vec{v}_1 \cdot \vec{p} = \vec{v}_1 \cdot (\alpha\vec{v}_1 + \beta\vec{v}_2) = \alpha v_1^2 + \beta \vec{v}_1 \cdot \vec{v}_2$$

but $v_1^2 = 1$ & $\vec{v}_1 \cdot \vec{v}_2 = \gamma$ (by assumption), so $= \alpha + \beta\gamma = \varnothing \Rightarrow \beta = -\alpha/\gamma$

& we have:

$$\vec{p} = \alpha(\vec{v}_1 - 1/\gamma\, \vec{v}_2)$$

& we can normalize $\vec{p}$:

$$p^2 = \alpha^2(\vec{v}_1 - 1/\gamma\,\vec{v}_2)\cdot(\vec{v}_1 - 1/\gamma\,\vec{v}_2)$$

$$= \alpha^2(1 - 2 + 1/\gamma^2)$$

$$= \alpha^2(1/\gamma^2 - 1)$$

& we want $p^2 = 1$, so

$$\alpha^2\left(\frac{1-\gamma^2}{\gamma^2}\right) = 1 \Rightarrow \boxed{\alpha = \frac{\gamma}{\sqrt{1-\gamma^2}}, \text{ then } \beta = \frac{-1}{\sqrt{1-\gamma^2}}}$$

$$\boxed{\vec{p} = \frac{\gamma}{\sqrt{1-\gamma^2}}\left[\vec{v}_1 - 1/\gamma\, \vec{v}_2\right]}$$

$\gamma - 1/\gamma$