Problem 25.1 (Griffiths 4.17)

a. $V(r) = -\frac{GMm}{r}$ & comparing w/ $V_e(r) = \frac{-e^2}{4\pi\epsilon_0}\frac{1}{r}$, we can take: $e^2 \rightarrow 4\pi\epsilon_0 GMm$ to make replacements.

b. $a^{-1} = \frac{me^2}{4\pi\epsilon_0 \hbar^2} \longrightarrow a_g^{-1} = \frac{GMm^2}{\hbar^2} \Rightarrow \boxed{a_g = \frac{\hbar^2}{GMm^2}}$

For $G = 6.67\times10^{-11}\ \frac{Nm^2}{kg^2}$, $M = 2\times10^{30}\ kg$, $m = 6\times10^{24}\ kg$ (from Giancoli)

then: $\boxed{a_g \approx 2.3\times10^{-138}\ m}$

c. $E_n = \frac{-\hbar^2}{2ma^2}\cdot\frac{1}{n^2} \longrightarrow \frac{-\hbar^2}{2ma_g^2}\frac{1}{n^2} \approx \boxed{-1.7\times10^{182}\ J/n^2}$

For a circular orbit, $\frac{mv^2}{R} = \frac{GMm}{R^2} \Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{2R}$

& the total energy is:

$$E = \frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{2R} = \frac{\hbar^2}{2ma_g}\cdot\frac{GMm^2}{\hbar^2}\cdot\frac{1}{n^2}$$

or $\frac{1}{n^2} = a_g/R \Rightarrow n = \sqrt{R/a_g} = \sqrt{\frac{150\times10^6\ km\cdot\frac{1000\ m}{1\ km}}{2.3\times10^{-133}\ m}} \approx \boxed{2.5\times10^{74}}$

d. $\Delta E = +1.7\times10^{182}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right] \approx +1.7\times10^{182}\left[\frac{1}{(n_i-1)^2} - \frac{1}{n_i^2}\right] \approx +1.7\times10^{182}\left[\frac{2}{n_i^3}\right] J \approx 2.2\times10^{-41}\ J$

$= \frac{1}{n_i^2}\left[\frac{1}{(1-1/n_i)^2} - 1\right] \approx \frac{1}{n_i^2}\left[1 + \frac{2}{n_i} - 1\right] = \frac{2}{n_i^3}$

$\Delta E = \hbar\omega \Rightarrow \omega = \frac{2.2\times10^{-41}\ J}{1.05\times10^{-34}\ Js} \approx 2.1\times10^{-7}\ 1/s$ so $\lambda = c\cdot T = c\cdot\frac{2\pi}{\omega} \approx \boxed{9\times10^{15}\ m}$

~1 light year.

Problem 25.2 (Griffiths 4.25)

For a rigid body, $L = I\omega$ & $I = \frac{2}{5}mr^2$, $\omega = \frac{v}{r}$, so

$$L = \frac{2}{5}mr^2\frac{v}{r}$$

$$= \frac{2}{5}mvr \Rightarrow v = \frac{5}{2}\frac{L}{mr}$$

for $L = \hbar/2$, $v = \frac{5\hbar}{4mr}$ w/ $r = \frac{e^2}{4\pi\epsilon_0 mc^2}$

$\approx \boxed{5.1\times10^{10}\ m/s.}$

Problem 25.3 (Griffiths 4.43)

a. $\psi_{321}(r,\theta,\phi) = R_{32}(r)\, Y_2^1(\theta,\phi)$

$$= \frac{-4}{81\sqrt{30}} \frac{r^2}{a^{7/2}} e^{-r/3a} \cdot \sqrt{\frac{15}{8\pi}} \sin\theta\cos\theta\, e^{i\phi}$$

$$= \boxed{-\frac{1}{81\sqrt{\pi}} \frac{r^2}{a^{7/2}} e^{-r/3a} \sin\theta\cos\theta\, e^{i\phi}}$$

b. $$\int_0^\infty \int_0^{2\pi} \int_0^{\pi} \frac{1}{81^2\pi} \frac{r^4}{a^7} e^{-2r/3a} \sin^3\theta \cos^2\theta\, r^2\, d\theta\, d\phi\, dr$$

$\int_0^\pi \sin^3\theta\cos^2\theta\, d\theta = 4/15$, $\int_0^{2\pi} d\phi = 2\pi$

$$= \frac{8}{81^2 \cdot 15} \frac{1}{a^7} \underbrace{\int_0^\infty r^6 e^{-2r/3a}\, dr}_{= \frac{98415}{8} a^7} = 1 \checkmark$$