Problem 20.1

For $|a\rangle$ & $|b\rangle$ we have:

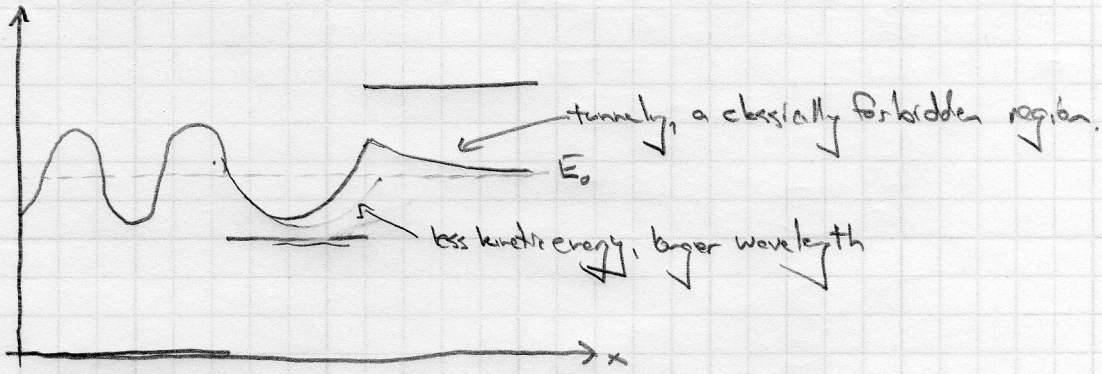
$$\langle a|a\rangle\langle b|b\rangle \geq |\langle a|b\rangle|^2$$

Take: $\langle f^2\rangle = \langle f|P\underbrace{P|f\rangle}_{\equiv|a\rangle} = \langle a|a\rangle$ & $\langle Q^2\rangle = \langle f|Q\underbrace{Q|f\rangle}_{\equiv|b\rangle} = \langle b|b\rangle$

then (*) reads:

$$\langle P^2\rangle\langle Q^2\rangle \geq |\langle f|PQ|f\rangle|^2 = |\langle PQ\rangle|^2.$$

Problem 20.2



Problem 20.3

a. $\psi(x) = \delta(x - x_0)$

b. $\langle p|\psi\rangle = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{+\infty} e^{-ipx/\hbar}\cdot\psi(x)\,dx$

$= \frac{1}{\sqrt{2\pi\hbar}} e^{-ipx_0/\hbar}$ a plane wave - not normalizable.

Problem 20.4

$$I = \int_0^{1/2a} \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{2\pi x}{a}\right)dx = -\frac{1}{4}\int_0^{1/2a}\left(e^{i\pi x/a} - e^{-i\pi x/a}\right)\left(e^{i2\pi x/a} - e^{-i2\pi x/a}\right)dx$$

$$= -\frac{1}{4}\int_0^{1/2a}\left[e^{i3\pi x/a} - e^{-i\pi x/a} - e^{i\pi x/a} + e^{-i3\pi x/a}\right]dx$$

$$= -\frac{1}{4}\int_0^{1/2a}\left[2\cos\left(3\pi x/a\right) - 2\cos\left(\pi x/a\right)\right]dx$$

$$= -\frac{1}{2}\left[\frac{a}{3\pi}\sin\left(\frac{3\pi x}{a}\right) - \frac{a}{\pi}\sin\left(\pi x/a\right)\right]\Big|_{x=0}^{1/2a}$$

$$= -\frac{a}{2\pi}\left[\frac{-1}{3} - 1\right] = \frac{2a}{3\pi}.$$

Problem 20.5

$A = \begin{pmatrix} p+iq & u+iv \\ s+it & x+iy \end{pmatrix}$ w/ all variables real

$A^\dagger = \begin{pmatrix} p-iq & s-it \\ u-iv & x-iy \end{pmatrix} = \begin{pmatrix} p+iq & u+iv \\ s+it & x+iy \end{pmatrix} \Rightarrow \begin{matrix} q=0 \\ y=0 \end{matrix}$ $s=u,\ t=-v$

So $A = \begin{pmatrix} p & u+iv \\ u-iv & x \end{pmatrix}$ & we get to choose $\{p, u, v, x\}$ 4 real numbers.