## Problem 13.1

We know that $\langle x\rangle = \frac{\hbar \bar{k} t}{m}$ & $\langle p\rangle = \hbar \bar{k}$, & we're given

$$\langle x^2\rangle = \frac{1}{4a} + \frac{a\hbar^2 t^2}{m^2} + \frac{\hbar^2\bar{k}^2 t^2}{m^2}$$

so $\boxed{\sigma_x^2 = \langle x^2\rangle - \langle x\rangle^2 = \frac{1}{4a}+\frac{a\hbar^2t^2}{m^2}}$

we could calculate $\sigma_p^2$ if we knew $\langle p^2\rangle$. We know that

$$\langle H\rangle = \frac{\hbar^2 a}{2m} + \frac{\hbar^2\bar{k}^2}{2m}$$

but $\langle H\rangle = \langle \frac{p^2}{2m}\rangle = \frac{1}{2m}\langle p^2\rangle$, so $\langle p^2\rangle = \hbar^2 a + \hbar^2\bar{k}^2$

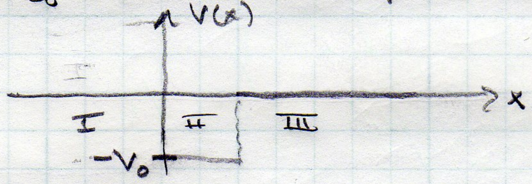
& $\boxed{\sigma_p^2 = \langle p^2\rangle - \langle p\rangle^2 = \hbar^2 a}$

The product is: $\sigma_x^2\sigma_p^2 = \left(\frac{1}{4a}+\frac{a\hbar^2t^2}{m^2}\right)(\hbar^2 a)$

$= \boxed{\frac{\hbar^2}{4} + \frac{\hbar^4 a^2 t^2}{m^2}}$ & this takes on its minimum value at $t=\emptyset$: ${\sigma_x^2\sigma_p^2}_{min} = \hbar^2/4$

## Problem 13.2 (Griffiths 2.40)

a. This potential partitions space into 3 segments:
$\infty$




We know that $\psi_I(x)=\emptyset$. In III we are solving: $-\frac{\hbar^2}{2m}\psi_{III}'' = -|E|\psi_{III}$
& set $k=\sqrt{\frac{2m|E|}{\hbar^2}}$, then

$$\psi_{III}(x) = Ae^{kx}+Be^{-kx}$$ & $x=\infty$ is in III, so $A=\emptyset$.

$$\psi_{III}(x) = Be^{-kx}$$

In region II, we have: $-\frac{\hbar^2}{2m}\psi_{II}'' - V_0\psi_{II} = -|E|\psi_{II}$

or $\psi_{II}'' = -\frac{2m}{\hbar^2}(-|E|+V_0)\psi_{II}$ & let $q^2 \equiv \frac{2m}{\hbar^2}(-|E|+V_0) = -k^2+\frac{2mV_0}{\hbar^2}$

then: $\psi_{II}(x) = A\sin(qx)+B\cos(qx)$

we have $\psi_I(\emptyset) = \psi_{II}(\emptyset) \Rightarrow \emptyset = B$, so $\psi_{II}(x) = A\sin(qx)$

& $\psi_{II}(a)=\psi_{III}(a) \Rightarrow$

$$A\sin(qa) = Be^{-ka}$$

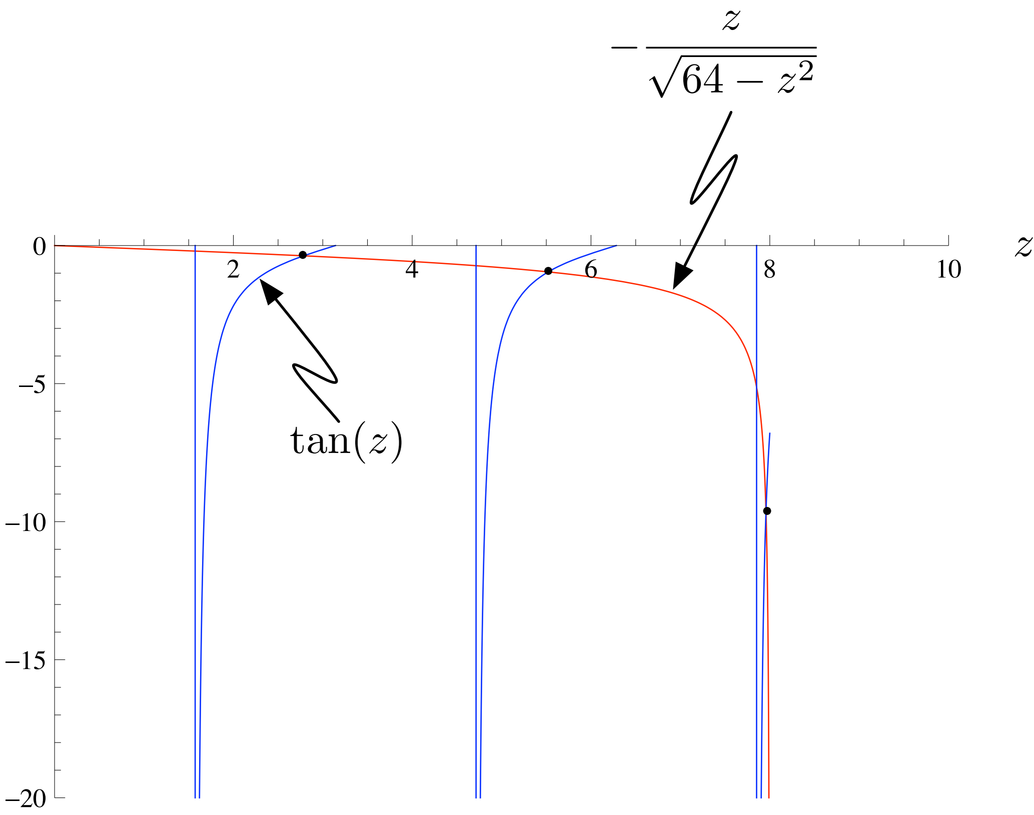
& $\frac{d\psi_{II}}{dx}\Big|_a = \frac{d\psi_{III}}{dx}\Big|_a \Rightarrow Aq\cos(qa) = -kBe^{-ka}$ $\Rightarrow \tan(qa) = -q/k = \frac{-q}{(64/a^2-q^2)^{1/2}}$

$z = qa$

so $\tan(z) = \frac{-z}{\sqrt{64-z^2}}$

attached is a plot of $\tan(z)$ vs. $-z/\sqrt{64-z^2}$, showing $\boxed{3}$ points at which holds.

dots indicate points of equality, where $\tan(z) = -\frac{z}{\sqrt{64-z^2}}$.



Problem 13.3

Set $\beta \equiv \frac{m\alpha}{\hbar^2 k}$, then the bound state is: $\psi_b(x) = \sqrt{\beta k}\, e^{-k\beta|x|}$
and the scattering states are:

$$\psi_k(x) = \begin{cases} A\left(e^{ikx} + \frac{i\beta}{1-i\beta} e^{-ikx}\right) & x \leq \emptyset \\ A(1-i\beta)^{-1} e^{ikx} & x \geq \emptyset \end{cases}$$

We want to show that $\int_{-\infty}^{+\infty} \psi_b(x)^* \psi_k(x)\,dx = \emptyset \quad \forall k$

The integral can be written:

$$\int_{-\infty}^{+\infty} \psi_b(x)^* \psi_k(x)\,dx = A\sqrt{\beta k}\Big[\underbrace{\int_{-\infty}^{\emptyset} e^{+k\beta x}\left(e^{ikx} + i\beta(1-i\beta)^{-1} e^{-ikx}\right)dx}_{I_1} + \underbrace{\int_0^{\infty} e^{-k\beta x}(1-i\beta)^{-1} e^{ikx}\,dx}_{I_2}\Big]$$

$$I_1 = \frac{1}{k(i+\beta)} + \frac{i\beta}{(1-i\beta)k(\beta-i)} \quad \text{\&} \quad I_2 = \frac{-1}{(1-i\beta)}\left[\frac{1}{k(i-\beta)}\right]$$

adding these two gives:

$$= \frac{A\sqrt{\beta k}}{k(i+\beta)}\left[1 - \frac{\beta}{\beta - i} - \frac{i}{i-\beta}\right]$$

$$= \frac{A\sqrt{\beta k}}{k(i+\beta)}\left[1 + \frac{\beta - i}{i-\beta}\right]$$

$$= \emptyset. \quad \checkmark$$