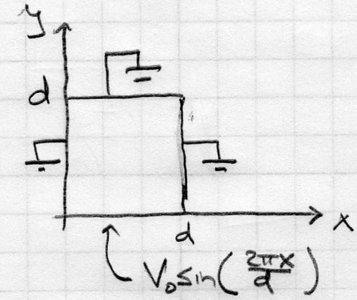
Problem 1.1



We have:

$\nabla^2 V = \varnothing$ inside, &

1. $V(\varnothing, y) = \varnothing$
2. $V(d, y) = \varnothing$
3. $V(x, d) = \varnothing$
4. $V(x, \varnothing) = V_0 \sin\left(\frac{2\pi x}{d}\right)$

Start w/ $V(x,y) = V_x(x) V_y(y)$, then:

$$\frac{\nabla^2 V}{V} = \underbrace{\frac{V_x''}{V_x}}_{\text{func. of } x} + \underbrace{\frac{V_y''}{V_y}}_{\text{func. of } y} = \varnothing$$

then $-\frac{V_x''}{V_x} = \alpha^2 = \frac{V_y''}{V_y} \Rightarrow$ $V_x(x) = A\cos(\alpha x) + B\sin(\alpha x)$

$V_y(y) = Ce^{\alpha y} + De^{-\alpha y}$

and the "general" solution is:

$$V(x,y) = [A\cos(\alpha x) + B\sin(\alpha x)][Ce^{\alpha y} + De^{-\alpha y}]$$

Now imposing boundary conditions:

1. $V(\varnothing, y) = A[Ce^{\alpha y} + De^{-\alpha y}] = \varnothing \Rightarrow \boxed{A = \varnothing}$
2. $V(d, y) = B\sin(\alpha d)[Ce^{\alpha y} + De^{-\alpha y}] \Rightarrow \alpha d = n\pi \Rightarrow \boxed{\alpha = \frac{n\pi}{d}}$

(for integer n)

3. $V(x, d) = B\sin(\alpha x)[Ce^{\alpha d} + De^{-\alpha d}] = \varnothing \Rightarrow \boxed{D = -Ce^{+2\alpha d}}$
4. $V(x, \varnothing) = BC\sin(\alpha x)[1 - e^{2\alpha d}] = V_0 \sin\left(\frac{2\pi x}{d}\right) \Rightarrow \boxed{BC = \frac{V_0}{1 - e^{2\alpha d}} \text{ and } n = 2}$

Then the final, unique solution is:

$$\boxed{V(x,y) = \frac{V_0}{1 - e^{4\pi}} \sin\left(\frac{2\pi x}{d}\right)\left[e^{\frac{2\pi y}{d}} - e^{-\left(\frac{2\pi y}{d} - 4\pi\right)}\right]}$$

Problem 1.2

The heat equation reads: $\frac{1}{k}\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$

a. Take $u(x,t) = u_x(x)\,u_t(t)$, then:

$$\frac{1}{k} u_t' u_x = u_t u_x''$$

$$\Updownarrow$$

$$\underbrace{\frac{1}{k}\frac{u_t'}{u_t}}_{\text{func. of } t} = \underbrace{\frac{u_x''}{u_x}}_{\text{func. of } x.}$$

Then each side must seperately equal (the same) constant, take

$$\frac{1}{k}\frac{u_t'}{u_t} = -\alpha^2 = \frac{u_x''}{u_x}$$

so we have: $u_t' = -\alpha^2 k u_t \Rightarrow u_t = Ae^{-\alpha^2 kt}$

& $u_x'' = -\alpha^2 u_x \Rightarrow u_x = B\cos(\alpha x) + C\sin(\alpha x)$

The "general" solution is: $\boxed{u(x,t) = Ae^{-\alpha^2 kt}\left[B\cos(\alpha x) + C\sin(\alpha x)\right]}$ (*)

b. We are given: 1. $u(\emptyset,t) = \emptyset$ 2. $u(d,t) = \emptyset$ 3. $u(x,\emptyset) = u_0\sin\left(\frac{\pi x}{d}\right)$

& applying these to (*) gives:

1. $u(\emptyset,t) = Ae^{-\alpha^2 kt}[B] = \emptyset \Rightarrow \boxed{B = \emptyset}$

2. $u(d,t) = Ae^{-\alpha^2 kt}[C\sin(\alpha d)] = \emptyset \Rightarrow \alpha d = n\pi \Rightarrow \boxed{\alpha = \frac{n\pi}{d}}$ for integer n.

3. $u(x,\emptyset) = AC\sin(\alpha x) = u_0\sin\left(\frac{\pi x}{d}\right) \Rightarrow \boxed{AC = u_0}$ and $\boxed{n = 1}$

So our solution is:

$$\boxed{u(x,t) = u_0 e^{-k\left(\frac{\pi}{d}\right)^2 t}\sin\left(\frac{\pi x}{d}\right)}$$