Final Exam

Quantum Mechanics I
Physics 342

Date: May 10th, 2010

NAME:

Summer address (if you want your exam sent)

There are five problems, each worth 10 points, the examination will be graded out of 50. Show as much work as you can for all problems. You have three hours, and can use the front and back of two 3×5 (inch) cards – the front and back covers of the book are provided separately.

Problem 1.1

$(10 = 3.\bar{3} + 3.\bar{3} + 3.\bar{3})$

a. What is the magnitude $|a|$ for the complex number $a = 5 + 3\,i$?

b. What is the decomposition of the function $f(x) = x\,(x-1)$ for $x = 0$ to 1 in terms of the sine series? That is, find the coefficients α_n in

$$f(x) = \sum_{n=1}^{\infty} \alpha_n \, \sin(n\,\pi x)\,. \tag{1}$$

c. You have a particle of spin 2 (the graviton) and an electron (spin one-half) – find the two-particle state, $|s\,m\rangle$, that is a simultaneous eigenstate of the total spin (squared) S^2 and S_z operators with $s = \frac{5}{2}$, $m = \frac{3}{2}$.[1]

[1]Remember that for a generic spin operator, with eigenstates $|s\,m\rangle$, we have the raising and lowering operators defined via:

$$S_{\pm}|s\,m\rangle = \hbar\,\sqrt{s\,(s+1) - m\,(m\pm 1)}\,|s\,(m\pm 1)\rangle. \tag{2}$$

Problem 1.2

$(10 = 3.\bar{3} + 3.\bar{3} + 3.\bar{3})$

a. Evaluate the commutator of L_z and p_y – what is $[L_z, p_y]$?

b. From the above, fill in the right-hand-side of

$$\sigma_{L_z}\,\sigma_{p_y} \geq \text{ ??} \tag{3}$$

c. What are the eigenvalues of the parity operator P, defined for functions of one-dimension as: $P\,f(x) = f(-x)$?

Problem 1.3

Find the energy spectrum of a particle under the influence of the one-dimensional Schrödinger equation for the harmonic oscillator potential with an energy offset V_0: $V(x) = \frac{1}{2}\, m\, \omega^2\, x^2 + V_0$ with $V_0 > 0$. The raising and lowering operators are defined to be:

$$a_\pm = \frac{1}{\sqrt{2\,\hbar\, m\, \omega}}\, (\mp i\, p + m\, \omega\, x)\,. \tag{4}$$

Problem 1.4

Consider the following Hydrogenic energy eigenstate:

$$\psi(\mathbf{r}) = F(r)\,\frac{1}{\sqrt{3}}\left(Y_1^0(\theta,\phi)\,\chi_+ + \sqrt{2}\,Y_1^1(\theta,\phi)\,\chi_-\right) \tag{5}$$

where $F(r)$ is the r-dependent portion of the wave-function (assume it has been normalized separately).

a. What is the minimum energy that could be returned by an energy measurement?

b. With what probability will a measurement of the electron's spin return $\frac{\hbar}{2}$?

c. The total spin operator is the sum $\mathbf{J} = \mathbf{L} + \mathbf{S}$, compute the expectation value: $\langle J_z \rangle$.

Problem 1.5

Two (non-interacting) electrons are in a two-dimensional infinite square "well" – the potential is:

$$V(x,y) = \begin{cases} 0 & 0 \le x \le a \text{ and } 0 \le y \le a \\ \infty & \text{else} \end{cases} \tag{6}$$

a. Find the lowest energy eigenstate for this system, and write its wavefunction.

b. You make an energy measurement of $E = 7\,\frac{\hbar^2\,\pi^2}{2\,m\,a^2}$ – after this measurement, the system is in a linear combination of states with the same energy – how many such states are there?