Mid-Term 2

Quantum Mechanics I
Physics 342

Date: April 9th, 2010

NAME:

There are three problems, each worth 10 points. Show as much work as you can for all problems. You have 50 minutes, and can use the front and back of a 3×5 (inch) card – the front and back covers of the book are provided separately.

Problem 2.1

(10 points, 2 per part)

a. Find the two solutions to the ODE:

$$\frac{d}{dr}\left(r^2\,\frac{df(r)}{dr}\right) = 2\,f(r). \tag{1}$$

b. In two dimensions, working in polar coordinates (s, ϕ), we can consider the infinite "round" well:

$$V(s) = \begin{cases} 0 & s < R \\ \infty & s > R \end{cases}. \tag{2}$$

In the plots below, circle the valid radial solution to this problem.

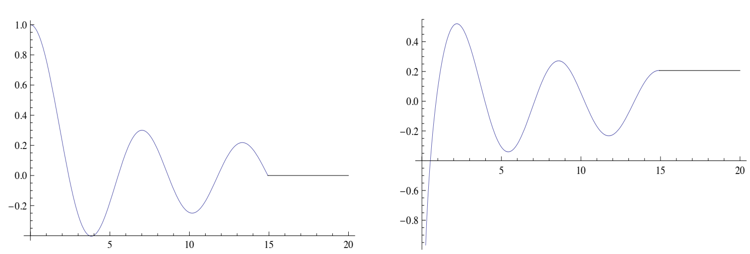

Figure 1: Candidate radial wave functions for two dimensional, polar coordinates.

c. As with the previous section, we are in two dimensions with (s, ϕ). This time, our well is finite:

$$V(s) = \begin{cases} 0 & s < R \\ V_0 & s > R \end{cases} . \tag{3}$$

In the plots below, circle the valid radial solution to the finite well problem.

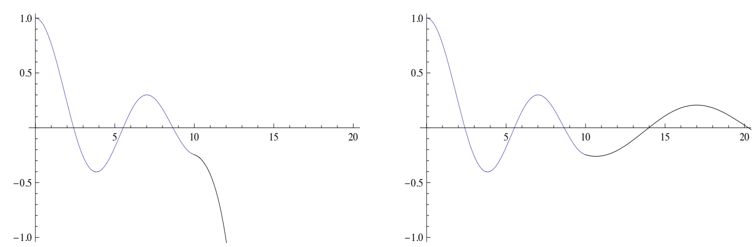

Figure 2: Candidate radial wave functions for two dimensional, polar coordinates.

d. Prove that the commutator of two Hermitian operators $\hat{A}$ and $\hat{B}$ is anti-Hermitian.

e. For some (one-dimensional) Hamiltonian $\hat{H}$ with stationary states $|\psi\rangle$, we know that:

$$\frac{d}{dt} \langle\psi| \, x \, p \, |\psi\rangle = 0. \tag{4}$$

Use this to show that $\langle x \, V' \rangle = 2 \langle T \rangle$ for these states (here, $V(x)$ is the potential, and $V' \equiv \frac{dV}{dx}$)[1].

[1]Remember that $\frac{d}{dt}\langle\hat{Q}\rangle = \frac{i}{\hbar}\langle[\hat{H},\hat{Q}]\rangle$ for an operator $\hat{Q}$.

Problem 2.2

(10 points)

A particle of mass m is constrained to move at a fixed radius a in two dimensions but is otherwise "free"[2]. If the system makes a transition from the first excited state to the ground state, emitting a photon ($E_\gamma = 2\,\pi\,\hbar\,\nu$) in the process, what is the wavelength of the emitted light? Leave your answer in terms of constants given in the problem and $\hbar$ and c.

[2]Hint: The constraint implies that the wave function can be viewed as a function only of the polar coordinate ϕ, since the location in s is known with certainty. Use this, and the polar Laplacian:

$$\nabla^2 f(s,\phi) = \frac{1}{s}\frac{\partial}{\partial s}\left(s\,\frac{\partial f}{\partial s}\right) + \frac{1}{s^2}\,\frac{\partial^2 f}{\partial \phi^2} \tag{5}$$

to determine the allowed energies – you must require that $\psi(\phi + 2\,\pi) = \psi(\phi)$.

Problem 2.3

(10 points)

Consider an infinite square well with potential $V_0 \sin(\omega\, t)$ inside the well, so that

$$V(x,t) = \begin{cases} V_0 \sin(\omega\, t) & 0 < x < a \\ \infty & x < 0 \text{ and } x > a \end{cases} . \tag{6}$$

Find $\Psi(x,t)$ if $\Psi(x,0) = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi\, x}{a}\right)$.