Mid-Term 1

Quantum Mechanics I
Physics 342

Date: March 5th, 2010

NAME:

There are four problems, each worth 10 points. Show as much work as you can for all problems. You have 50 minutes, and can use the front and back of a 3×5 (inch) card – the front and back covers of the book are provided separately.

Problem 1.1

a. For the attached movie, sketch a plausible potential, describe your motivation. There are infinite walls on the left and right indicated in blue (?)

b. Mark the (approximate position of the) point $z = i^i$ in the complex plane shown below. The unit circle is shown for reference.

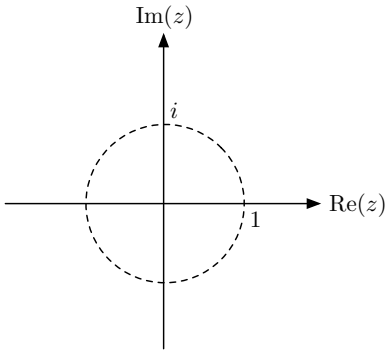

Figure 1: Unit circle in the complex plane.

c. You are given the initial state $\bar{\psi}(x) = A\,x\,(x-a)$ for the infinite square well potential, with $V(0) = V(a) = 0$. Find the probability that a particle is initially between $x = 0$ and $x = \frac{1}{2}\,a$.

Problem 1.2

Find the stationary states of the infinite step potential:

$$V(x) = \begin{cases} 0 & x < 0 \\ \infty & x > 0 \end{cases} \tag{1}$$

Problem 1.3

For the infinite square well with $V(0) = V(a) = \infty$, a particle of mass m starts in the initial state:

$$\bar{\psi}(x) = A\left(\psi_1(x) + 2\,\psi_2(x)\right). \tag{2}$$

a. What outcomes are possible for a measurement of the particle's energy, and what is the probability of each?

b. What is the expectation value of position (as a function of time)? Note: This problem and the next are far more time-consuming than the actual midterm problems – they are meant to provide additional conceptual practice. In an actual midterm, you would only be asked to setup the relevant expressions, ignoring the computational details.

Problem 1.4

You measure a particle in the ground state of the SHO ($V(x) = \frac{1}{2}\, m\, \omega^2\, x^2$). At time $t = 0$, you turn off the potential, wait Δt, then turn it back on. What is the probability of measuring the particle energy to be $\frac{1}{2}\, \hbar\, \omega$?