Statistical Interpretation II

Lecture 19

Physics 342
Quantum Mechanics I

Wednesday, March 10th, 2010

We'll finish with some final comments on the statistical interpretation of kets like $|\Psi\rangle$, both in discrete (but infinite) vector spaces and continuous infinite spaces. In particular, we'll discuss the role of commutators and observables, and look at the implications of vanishing commutator for observables.

Homework

The reading is meant to solidify some of the Bra-ket ideas from class – we'll return to uncertainty principles on Friday.

Reading: Griffiths, pp. 118–123.

Problem 19.1

Consider the two matrices:

$$\mathbb{A} \doteq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \mathbb{B} \doteq \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \tag{19.1}$$

These clearly commute: $[\mathbb{A}, \mathbb{B}] = 0$, since $\mathbb{A}$ is the identity matrix. This problem is meant to show you that we cannot take just *any* eigenvectors of $\mathbb{A}$ and expect them to be eigenvectors of $\mathbb{B}$.

a. Show that $\mathbf{v}_1 \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{v}_2 \doteq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are eigenvectors of $\mathbb{A}$, and find the eigenvalues. Are these eigenvectors of $\mathbb{B}$?

b. Find the eigenvectors and eigenvalues of $\mathbb{B}$, show that *these are* eigenvectors of $\mathbb{A}$.

Conclusion: We can *find* simultaneous eigenvectors of $\mathbb{A}$ and $\mathbb{B}$ (since they commute), but we are not guaranteed that any set of eigenvectors are simultaneous eigenvectors.

Problem 19.2

Show that an operator $\hat{Q}$ that is *anti*-Hermitian:

$$\int_{-\infty}^{\infty} \psi^*(x)\, \hat{Q}\, \psi(x)\, dx = -\int_{-\infty}^{\infty} \left(\hat{Q}\, \psi(x)\right)^* \psi(x)\, dx \tag{19.2}$$

has purely imaginary expectation values: $\langle \hat{Q} \rangle$ is imaginary.

Problem 19.3

Our choice of basis is important when computing expectation values (although, of course, the expectation value itself is independent of the basis in which we compute it), and, even earlier, when we solve Schrödinger's equation (think of the position versus momentum representation). Suppose we take the abstract form of Schrödinger's equation:

$$\hat{H}\ |\Psi(t)\rangle = i\,\hbar\,\frac{\partial}{\partial t}\ |\Psi(t)\rangle \tag{19.3}$$

and expand $|\Psi(t)\rangle$ in the natural basis defined by $\hat{H}$ (i.e. it's eigenstates).

a. Assume (because $\hat{H}$ is Hermitian) that we have a complete, continuous set of eigenstates of $\hat{H}$ indexed by k:

$$\hat{H}\ |\psi_k\rangle = E(k)\ |\psi_k\rangle \tag{19.4}$$

where $E(k)$ is a real number that is a function of k. In addition, we know that $\langle\psi_{k'}|\psi_k\rangle = \delta(k-k')$. The expansion, as a function of time of a general state $|\Psi(t)\rangle$ can be accomplished using the time-dependent decomposition coefficients $\phi(k,t)$ in:

$$|\Psi(t)\rangle = \int_{-\infty}^{\infty} \phi(k,t)\ |\psi_k\rangle\ dk \tag{19.5}$$

(so that the decomposition varies in time, but at each time t, we can represent the ket $|\Psi(t)\rangle$ in terms of the basis $|\psi_k\rangle$). Input this into (19.3), and simplify.

b. Now hit both sides of your result from part a. with $\langle\psi_{k'}|$ and use the orthonormality relation to solve for $\phi(k',t)$.