The Harmonic Oscillator III

Lecture 10

Physics 342
Quantum Mechanics I

Monday, February 15th, 2010

Today, we will finish our discussion of the harmonic oscillator. Our model is a massive particle in an infinite potential well, and we currently have the ground state and a prescription for obtaining higher energy states:

$$\begin{aligned}\psi_0(x) &= \left(\frac{m\,\omega}{\pi\,\hbar}\right)^{\frac{1}{4}}\, e^{-\frac{m\,\omega}{2\,\hbar}\,x^2}\\ \psi_n(x) &= \frac{1}{\sqrt{n!}}\,(a_+)^n\;\psi_0(x)\\ a_+ &\equiv \frac{1}{\sqrt{2\,m\,\hbar\,\omega}}\,(-i\,p + m\,\omega\,x)\,.\end{aligned} \tag{10.1}$$

The funny factor of $\sqrt{1/n!}$ comes from our desire to preserve the normalization of the ground state in the excited states. As for the energy, we have learned that the spectrum is linearly increasing with n via:

$$E_n = \left(n + \frac{1}{2}\right)\,\hbar\,\omega. \tag{10.2}$$

The usual story now holds – we have the "stationary" states, and these solve the time-independent Schrödinger equation. We can put these together with their time-dependent factors to generate linear combinations that solve the full Schrödinger equation:

$$\Psi(x,t) = \sum_{j=0}^{\infty} A_j\,\psi_j(x)\,e^{-i\,\frac{E_j\,t}{\hbar}} \tag{10.3}$$

where the coefficients $\{A_j\}_{j=0}^{\infty}$ are set by the initial wavefunction $\Psi(x,0)$ which must be provided. We rely on completeness (any function can be decomposed into these stationary states) and orthonormality (as we shall see) to actually compute the coefficients.

Homework

Reading: Griffiths, pp. 40–51.

Problem 10.1

We know the stationary states of the infinite square well that is centered at $a/2$ (so that $V = \infty$ for $x < 0$ and $x > a$, and zero in between). Find the stationary states (and energies) of the "symmetric" square well (centered at $x = 0$) governed by the potential:

$$V(x) = \begin{cases} \infty & x < -\frac{1}{2}\,a \text{ and } x > \frac{1}{2}\,a \\ 0 & -\frac{1}{2}\,a \le x \le \frac{1}{2}\,a \end{cases} \quad . \tag{10.4}$$

Problem 10.2

A particle is in an infinite square well (centered at $x = 0$ as in the previous problem). We make an energy measurement, and find:

$$E = \frac{\pi^2\,\hbar^2}{2\,m\,a^2}. \tag{10.5}$$

After this measurement, we turn off the infinite square well potential and turn on a harmonic confining potential ($V(x) = \frac{1}{2}\,m\,\omega^2$). What is the probability that we then measure the energy to be

$$E = \frac{1}{2}\,\hbar\,\omega\,? \tag{10.6}$$

It is not necessary to perform the tricky integration that shows up – just set up the integral.