The Harmonic Oscillator III

Lecture 10

Physics 342
Quantum Mechanics I

Monday, February 15th, 2010

Today, we will finish our discussion of the harmonic oscillator. Our model is a massive particle in an infinite potential well, and we currently have the ground state and a prescription for obtaining higher energy states:

$$\psi_0(x) = \left(\frac{m \omega}{\pi \hbar}\right)^{\frac{1}{4}} e^{-\frac{m \omega}{2\pi} x^2}$$

$$\psi_n(x) = \frac{1}{\sqrt{n!}} (a_+)^n \psi_0(x)$$

$$a_+ \equiv \frac{1}{\sqrt{2 m \hbar \omega}} (-i p + m \omega x).$$

(10.1)

The funny factor of $\sqrt{1/n!}$ comes from our desire to preserve the normalization of the ground state in the excited states. As for the energy, we have learned that the spectrum is linearly increasing with $n$ via:

$$E_n = \left(n + \frac{1}{2}\right) \hbar \omega.$$  

(10.2)

The usual story now holds – we have the “stationary” states, and these solve the time-independent Schrödinger equation. We can put these together with their time-dependent factors to generate linear combinations that solve the full Schrödinger equation:

$$\Psi(x, t) = \sum_{j=0}^{\infty} A_j \psi_j(x) e^{-i \frac{E_j t}{\hbar}}$$

(10.3)

where the coefficients $\{A_j\}_{j=0}^{\infty}$ are set by the initial wavefunction $\Psi(x, 0)$ which must be provided. We rely on completeness (any function can be decomposed into these stationary states) and orthonormality (as we shall see) to actually compute the coefficients.
Homework

Reading: Griffiths, pp. 40–51.

Problem 10.1

We know the stationary states of the infinite square well that is centered at $a/2$ (so that $V = \infty$ for $x < 0$ and $x > a$, and zero in between). Find the stationary states (and energies) of the “symmetric” square well (centered at $x = 0$) governed by the potential:

$$V(x) = \begin{cases} \infty & x < -\frac{1}{2}a \text{ and } x > \frac{1}{2}a \\ 0 & -\frac{1}{2}a \leq x \leq \frac{1}{2}a \end{cases}. \quad (10.4)$$

Problem 10.2

A particle is in an infinite square well (centered at $x = 0$ as in the previous problem). We make an energy measurement, and find:

$$E = \frac{\pi^2 \hbar^2}{2ma^2}. \quad (10.5)$$

After this measurement, we turn off the infinite square well potential and turn on a harmonic confining potential ($V(x) = \frac{1}{2}m\omega^2$). What is the probability that we then measure the energy to be

$$E = \frac{1}{2} \hbar\omega? \quad (10.6)$$

It is not necessary to perform the tricky integration that shows up – just set up the integral.