Minimizing Repeaters Through Probabilistic Methods

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Problem Statement: Part 1

One thousand radio users inside of a 40 mile radius region wish to communicate using relatively low-powered equipment. In order to communicate across the range, an installment called a "repeater" will be used. These repeaters receive low-power signals and transmit them at increased power on a different frequency. In this problem, the transmitter frequency is either 600 kHz above or below the receiver frequency.

Problem Statement: Part 2

Unfortunately, there is a limited range of MHz which can be used by this system (145 to 148 MHz). Because different channels must be a certain "distance" from each other in the frequency space, this limits the number of transmissions in a small area. Fortunately, this number can be increased by a continuous tone-coded squelch system (CTCSS). This system allows for multiple signals in the same frequency range without interference. In this problem, the number of signals that can share a frequency is 54.

Problem Statement: Additional Constraints

We should expect that changing the problem will result in a different solution. It therefore makes sense to test our hypothesized solution with different parameters. In this case, we analyzed the case where there are ten thousand users in the region, and the case where the region is mountainous.

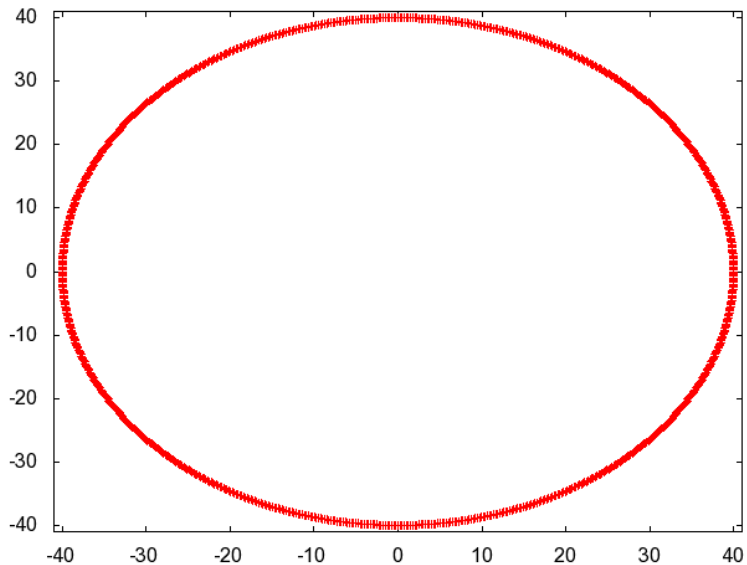
Simplifying Assumptions

Given the open-ended nature of the problem, we made a few assumptions about the problem:

* Radio users have a range of 15 miles.
* The range of a repeater is greater than 80 miles, but the power level can be adjusted to reduce this range.
* Every user contacts another at random.
* There must be 0.2 MHz between frequencies.
* At maximum, this allows for $54 * 7 = 378$ repeaters in the same range.

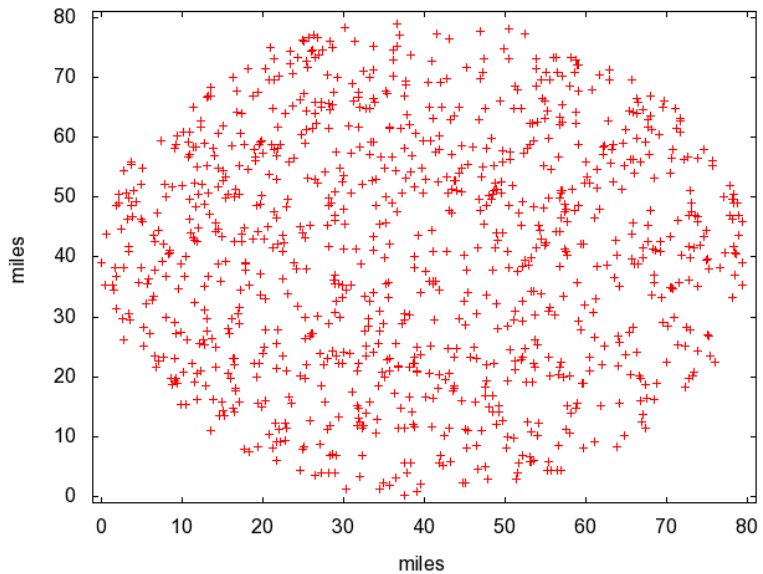
Worst Case Scenario

The worst case scenario is when all 1000 users are arranged in communicating pairs so as to maximize the distance between them; in other words, when they are diametrically arranged across the circle. This requires that each individual has a repeater, and these repeaters are bound to interfere with one another in a way which makes a solution impossible. Fortunately, this situation is implausible in the extreme.



Reduced Problem Space

Because finding a solution is not possible for every arrangement of users, we area bound to consider some other set of problems. In this case, we allow the users to be arranged randomly in the 40 mile radius. This makes the occurrence of unsolvable scenarios minimal. One possible scenario is below:



Lower-Bound Algorithm

With the aim of having an estimate on how many repeaters would be necessary to solve this problem, we created an algorithm which would, on average, yield a lower bound on how many repeaters are necessary. The algorithm is as follows:

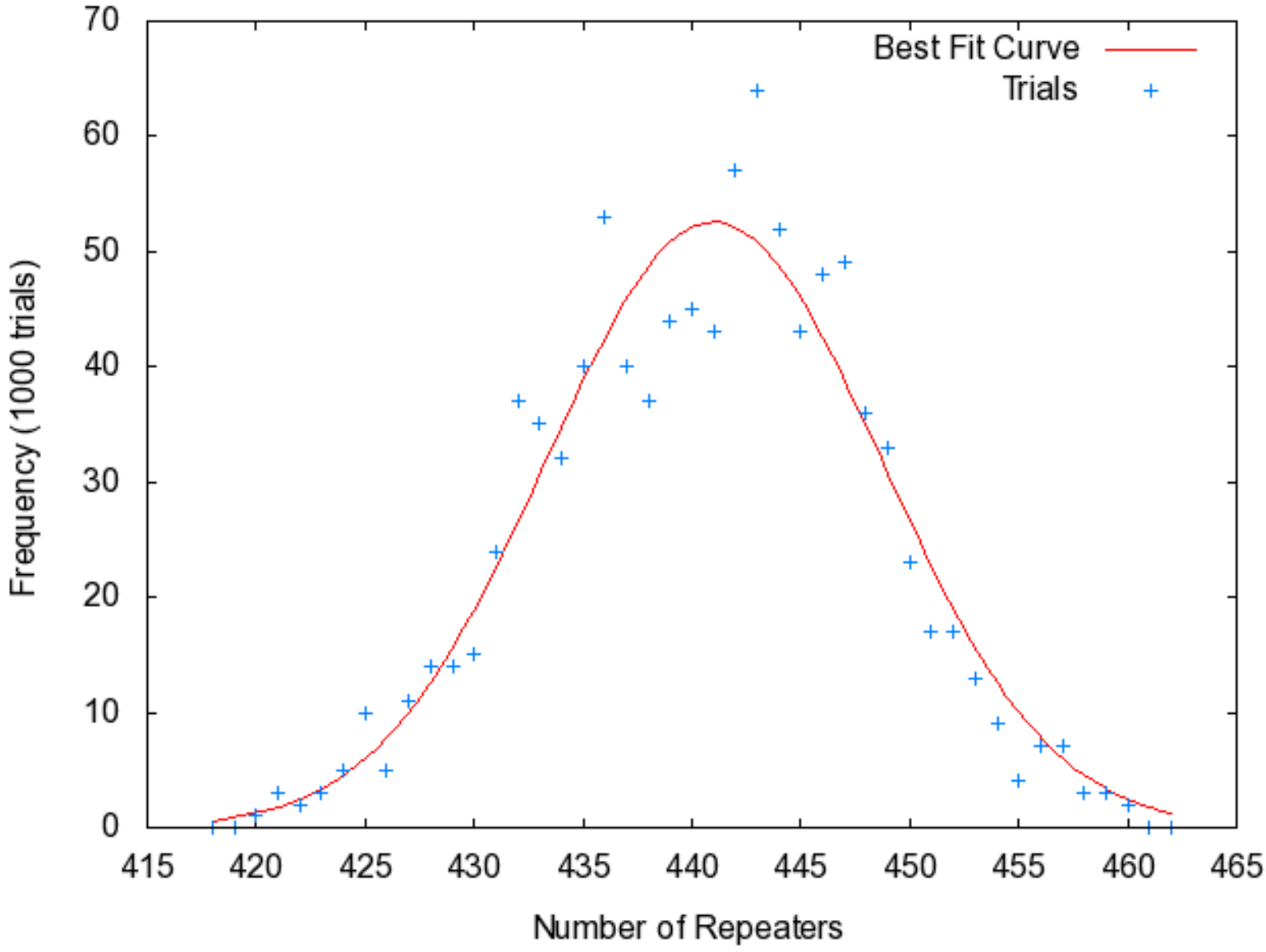
(1) Randomly select positions in the disk of radius 40 for each of the users.

(2) Pair each user with one other user to engage in conversation.

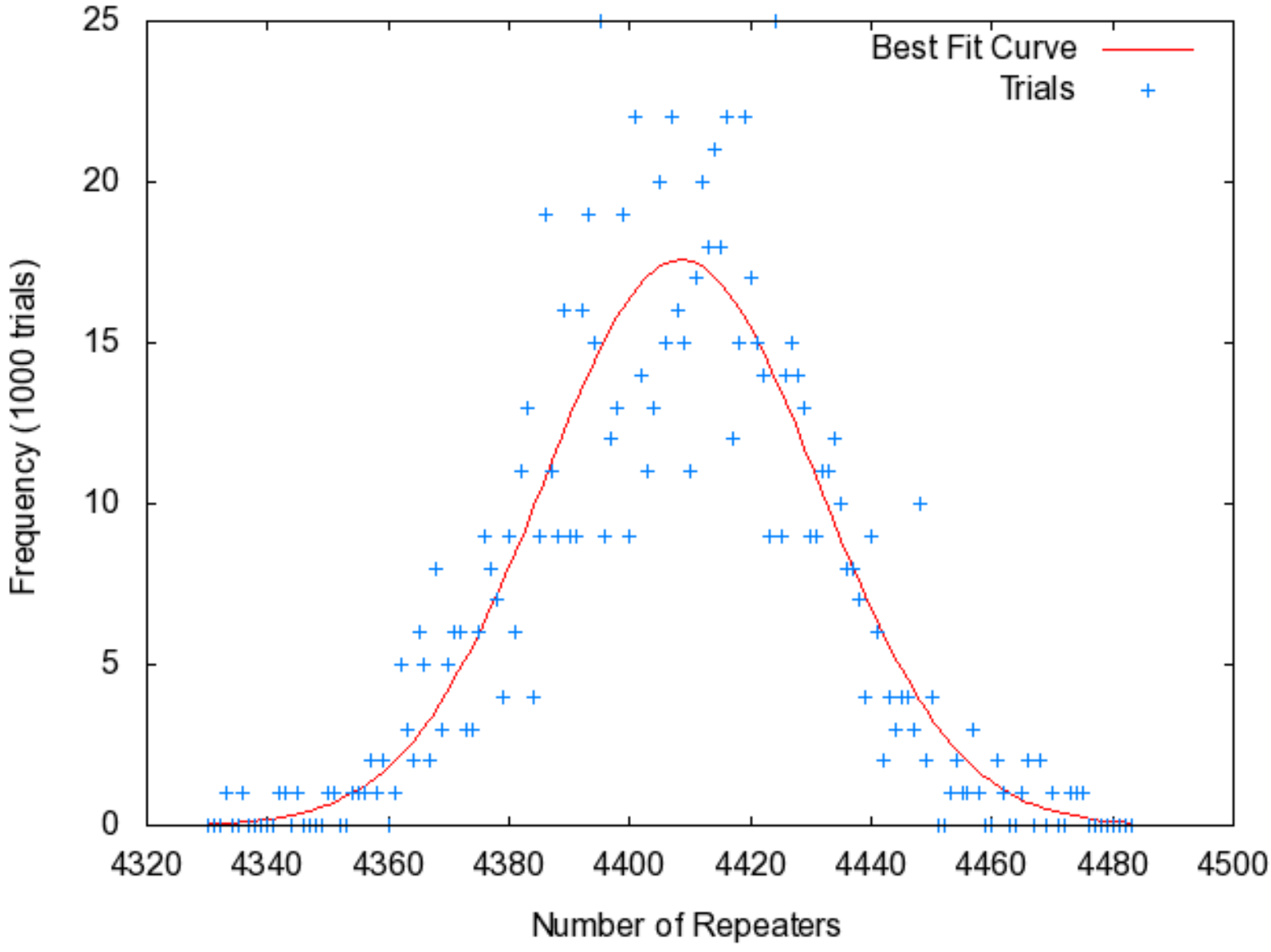
(3) Create a repeater for each conversation which cannot be successfully carried out. The power level for this receiver is tailored to the two users, and the position of the repeater is selected from a grid of possible locations. Assign each repeater a frequency pair which does not interfere with pre-existing signals in the area.

(4) Count the number of repeaters which was required. If it was impossible to assign some repeater a frequency-PL tone combination, then print a message to this effect.

Results for 1000 users



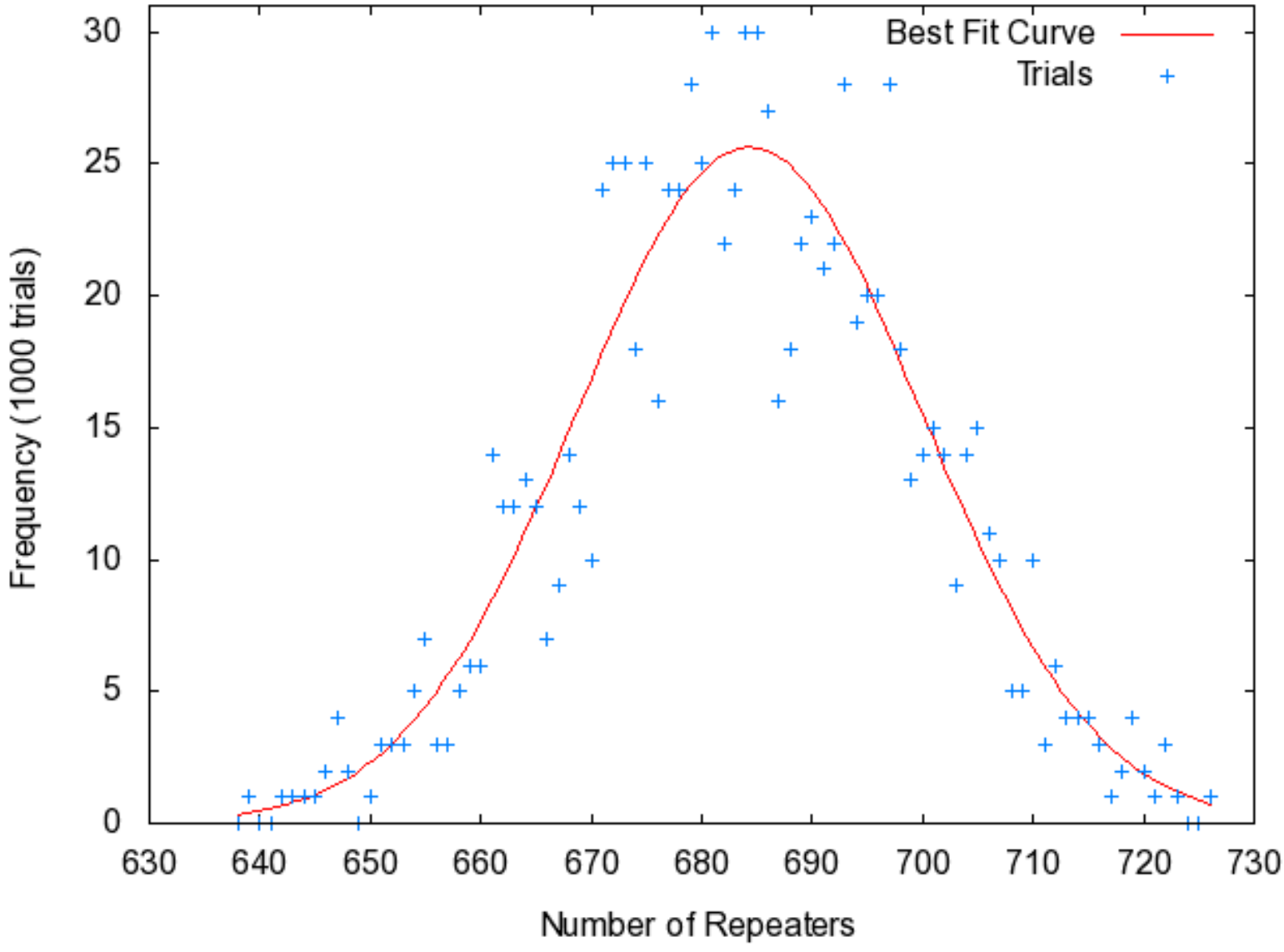
Results for 10,000 users



Modifications for Mountainous Regions

The disk on which the users communicated was divided by a single line. Users could transmit to other users lying on the same side of the line, but if two users wanted to communicate over the mountain range, a repeater would need to be added at the peak, in addition to other repeaters which might be needed to boost the signal in order to reach the repeaters.

Results for Mountainous Regions



Final Thoughts